A LITERATURE REVIEW OF INVISIBILITY (CLOAKING) DEVICES

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For the Pleasure of …..

and for Christine and Tony

ABSTRACT

An outline of the theoretic and experimental work concerning cloaking devices to the date of this paper is examined together with a suggestion for cloaking a ship.

Key words: Metamaterials, transformation optics, cloaking device, perfect lens, slab, antennae.

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SECTION 1. INTRODUCTION

For decades, science fiction writers have authored stories where characters employ faster than light propulsion systems, time travel and other devices we can only dream about. The ‘cloaking device’ and ‘personal shield’ have appeared in stories by Isaac Asimov (personal shield), Star Trek (cloaking and shields) and Harry Potter (magic invisibility cloak), to name just three.

The US Navy wanted their ships to be radar invisible to Japanese and German war machines and that culminated in the so-called ‘Philadelphia Experiment’. As that topic is so widely reported and commented about on the internet, nothing further will be said about it here, although anyone reading the reports and comments for the first time will be startled by the description of what was supposed to have happened to the U.S.S. Eldridge and its crew.

The purpose of this paper is to review the progress made so far. The remainder of this introduction briefly examines the properties of metamaterials and sections 2 to 5 deal with only the basics of invisibility cloaking. Section 6 sets out our ideas for a practical application and the Appendix contains references to further aspects.

To be completely effective, an invisibility cloak should operate at all wavelengths of electromagnetic radiation. Light should not be absorbed by the cloak because that would betray the cloaked object as a patch of blackness. A true cloak should bend light around the object cloaked. All materials interact with electromagnetic fields, including light. Therefore, materials such as an ordinary spectacle lens can control light and by altering say, the shape of the lens, light will be controlled differently. A lens merely beds light in a certain way and how light is bent depends on the properties of the material of which the lens is made. However, there is a limit to the capabilities of ordinary materials to affect light in this way and ‘capability’ is no more than talking about the electromagnetic properties of the lens material.

When an electromagnetic wave enters the lens, its fields interact with the charges of the particles in the substance of which the lens is made. In turn, that interaction changes the speed or wavelength (or both) of the wave. The fields are the electric and magnetic, both of which appear as coupled fields in Maxwell’s equations. From the viewpoint of an electromagnetic wave, the material it interacts with is describable *only* by its electric permittivity, (μ), and its magnetic permeability, (ε). Consider the possible alternatives of these in materials:

Category 1: μ > 0, ε > 0, being most known materials, natural or otherwise.

Category 2: μ > 0, ε < 0, being materials not well investigated.

Category 3: μ < 0, ε > 0, also being materials not well investigated.

Category 4: μ < 0, ε < 0 where these materials do not naturally exist.

Metamaterials (MMs) are man made and have properties directly related to their structure rather than their composition. Walser defined MMs as macroscopic composites having a man made, three-dimensional, periodic cellular architecture designed to produce an optimized combination, not available in nature, of two or more responses to specific excitation. That means an MM will affect electromagnetic waves if its structural features are smaller (usually around half or smaller) than the wavelength, ranging from about 250nm for visible light to one decimetre for microwaves. MMs usually consist of periodic structures (like photonic crystals and frequency selective surfaces), and to be effective, MMs should be homogeneous whereas, photonic crystals are not. Development of devices have been hampered by light loss through absorption by the metallic parts of the MM. That has now been solved by the addition of optical gain to the dielectric adjacent to the metal to compensate for lost light.

It is possible to create structures that demonstrate negative refractive indices and negative permittivity and negative permeability. For transparent natural materials, $n^2 = \pm\varepsilon\mu$ and all known natural materials have a positive refractive index. Some manufactured MMs have $\varepsilon < 0$ and $\mu < 0$ and because $\varepsilon\mu$ is positive, so is n. Veselago showed that materials with a negative $\sqrt{n}$ are capable of transmitting light but real materials will have complex-valued permeability and permittivity. Here, to display a negative refractive index, the real parts of permittivity and permeability do not need to be negative. Consequences arising from a negative refractive index material is that the Doppler shift is reversed, Cerenkov radiation has its direction reversed, wavefronts move opposite in direction to the energy flow (anti-parallel time-averaged Poynting vector) but Snell’s Law applies in the usual way. Since one refractive index is positive (say, air) and the other is negative (a MM), refraction will be reversed and be on the same side of the normal as the ray entered. For plane waves, the electric and magnetic fields as well as the wave vector are subject to a left-handed rule.

The first negative refractive index lens operating at microwave frequencies was demonstrated by Grbic and Elefththeriades and provided a resolution three times better than the diffraction limit. MMs have been proposed as the basis for a cloaking device that bends light around it. In 2006, an MM was made that rendered an object invisible to microwave radiation. It has been theorised that MMs could be built to bend matter around them, for example, to bend the trajectory of a bullet. An electromagnetic invisibility cloak requires the use of an anisotropic transform medium whose permittivity and permeability are obtained from a homogeneous isotropic medium, by transformations of coordinates [1]. Anisotropic materials with desired permittivity can be produced using a layered structure of alternating and thin dielectric layers or, metal and dielectric layers. These planar systems have been proposed to demonstrate sub-wavelength imaging and to build photonic funnels for sub-diffraction light compression and propagation. An ‘optical hyperlens’ of a cylindrical structure of anisotropic medium was proposed that should have the capability of far-field imaging with resolution below the diffraction limit. Such a hyperlens has been experimentally demonstrated and uses concentric alternating layers of metal and dielectric.

A method was proposed to achieve the required radius-dependent anisotropic material parameters and to construct an optical range electromagnetic cloak using a concentric alternating thin layered structure of homogeneous isotropic materials. With proper design of the permittivity or the thickness ratio of the alternating layers, the low-reflection and power-flow bending properties of the structure through rigorous analysis of the scattered electromagnetic fields can be shown. The proposed cloaking structure does not require anisotropy or inhomogeneity of the material constitutive parameters usually achieved with MMs with sub-wavelength structured inclusions in the form of split-ring resonators. In any case, the latter would not function at optical wavelengths [2]. Another proposal, [3], was that optical cloaking could be achieved with MMs having sub-wavelength inclusions of radially directed metal wires embedded in the dielectric material.

This may sound as if a cloak demands that spacetime be curved around a person or vehicle, that is, extremely locally. Indeed, if that could be achieved, the effect would be that of a cloak but the energy requirement to accomplish that feat would be huge. What has been proposed in the last few years does not affect spacetime curvature at all. Without altering spacetime curvature, all that needs to be done is take a volume of space in which there is a visible object because it reflects and scatters light and then, alter that volume of space in such a way that light from distant sources is bent around the boundary of the volume containing the object. That will make the object invisible. One way to back calculate the properties of the MM that will produce the desired effect is to solve the original coordinates in the transformed region using Laplace’s equations to numerically establish the deformation field. In turn the material properties of the devices to be designed follow from that [4].

Another way, [5], is to imagine there are no practical limits on the electromagnetic properties of materials. Given a desired function, how do we find the design of the device that turns this function into fact? According to Fermat's Principle, light rays follow the shortest optical paths in media and so are, effectively, geodesics and General Relativity deals with the theoretical tools for fields in curved geometries. Electromagnetic metamaterials, not masses, create effective geometries and the theory generalizes the concept behind the proposed perfect invisibility devices to magneto-electric or moving media and incorporates negative refraction. Then unify and generalize a range of physical phenomena that rely on the geometry of media.

These are many ways to back calculate the properties of the material needed to produce invisibility and what follows deals with some of those. Here, the word 'transformation' is coupled with the words 'optics', 'coordinates', 'media' and others. In this paper, it means that a material is used to alter (transform) a volume of space and everything associated with it.

The next four sections examine fundamental ideas.

SECTION 2. THEORY

Transformation optics is a simple approach to the design of MMs to alter the trajectories of electromagnetic waves passing through a volume of space, even though those trajectories conform to the local metric. Once the MM design is found, the coordinate transformation and its Jacobi matrix govern the transformation of Maxwell's equations. The altered (transformed) volume needs to be identical to a volume of free space if that replaced the transformed volume. With this method, anisotropy is required and any constraints placed on the MM used will reduce its performance for perfect invisibility cloaking. As set out in [6], denote Maxwell's equations in the Minkowski form

$$\partial_{[\kappa}F_{\lambda\nu]} = 0$$

$$\partial_{\nu}G^{\nu\lambda} = j^{\lambda}$$

$$\kappa, \lambda, \nu = 0,\ 1,\ 2,\ 3$$

where the square brackets express an alternation among the indices and the skew-symmetric tensor $F_{\lambda\nu}$ is related to the electric field and magnetic induction, the skew-symmetric tensor $G^{\nu\lambda}$ is related to the electric displacement and the magnetic field, and, the contravariant vector j^{λ} is related to the volume of charge density with j as the current density. In this notation, the coordinate vector in four-space is $x^{\alpha} = (x_0 = t,\ x_1,\ x_2,\ x_3)^T$ with c = 1. For a linear medium, the constitutive relation can be written as

$$G^{\lambda\nu} = \frac{1}{2}\chi^{\lambda\nu\sigma\kappa}F_{\sigma\kappa}$$

where the tensor $\chi^{\lambda\nu\sigma\kappa}$ contains complete information about the electro-magnetic material properties. The Minkowski equations and the constitutive relation are form-invariant for arbitrary continuous space-time transformations in the form

$$x^{\alpha'}(x^{\alpha}) = \frac{\partial x^{\alpha'}}{\partial x^{\alpha}}x^{\alpha} = A_{\alpha}^{\alpha'}x^{\alpha}$$

where $A_{\alpha}^{\alpha'}$ are the elements of the Jacobian transformation matrix and the primed indices denote the space-time coordinates of the vector x in the transformed space. In fact, that equation characterizes the geometric variation between the original space and the distorted space and the determination of ‘A’ is crucial in the design of the transformation medium. For that transformation, the Minkowski equations transform as

$$\partial_{[\kappa'}F_{\lambda'\nu']} = 0$$
$$\partial_{\nu'}G^{\nu'\lambda'} = j^{\lambda'}$$

and the constitutive relation transforms as

$$G^{\lambda'\nu'} = \frac{1}{2}\chi^{\lambda'\nu'\sigma'\kappa'}F_{\sigma'\kappa'}$$

with

$$\chi^{\lambda'\nu'\sigma'\kappa'} = \left[\det(A_{\lambda}^{\lambda'})\right]^{-1}A_{\lambda}^{\lambda'}A_{\nu}^{\nu'}A_{\sigma}^{\sigma'}A_{\kappa}^{\kappa'}\chi^{\lambda\nu\sigma\kappa}$$

$$F_{\sigma'\kappa'} = A_{\sigma'}^{\sigma}A_{\kappa'}^{\kappa}F_{\sigma\kappa}$$

The form-invariance of the Minkowski equations and the constitutive relation also hold for transformations that address only the space coordinates, because the space manifold is a sub-manifold of the spacetime manifold.

Now restrict things to time-independent spatial coordinate transformations. Under this restriction, the constitutive parameters, i.e. the tensors of the permittivity and the permeability of a linear, anisotropic, non-dispersive, non bi-anisotropic and locally interacting medium can be written in a more accessible form as

$$\varepsilon^{i'j'} = \left[\det\left(A^{i'}_{\ i}\right)\right]^{-1} A^{i'}_{\ i} A^{j'}_{\ j} \varepsilon^{ij} \quad \text{and} \quad \mu^{i'j'} = \left[\det\left(A^{i'}_{\ i}\right)\right]^{-1} A^{i'}_{\ i} A^{j'}_{\ j} \mu^{ij} .$$

These last two relationships together with $x^{\alpha'}(x^{\alpha}) = A^{\alpha'}_{\alpha} x^{\alpha}$ form the underlying equations for the calculation of the electromagnetic material parameters used in the design of a square-shaped cloak and a concentrator for electromagnetic fields.

Space transformation can be viewed as the deformation of a material and the permittivity and permeability tensors in the transformed space are found to correlate with the deformation field of the material. By solving the Laplace equation that describes how the material will deform during a transformation, an electromagnetic cloak with arbitrary shape can be designed if the boundary conditions of the cloak are considered. The material parameters of spherical and elliptical cylindrical cloaks are derivable based on the analytical solutions of the Laplace equation. For cloaks with irregular shapes, the material parameters of the transformation medium are determined numerically by solving that equation.

In continuum mechanics, the tensor A is called the deformation gradient tensor for an infinitesimal element deformed under the space transformation. The transformation can be decomposed into a pure stretch deformation (described by a positive definite symmetric tensor, **V**) and a rigid body rotation (described by a proper orthogonal tensor, **R**), so the tensor can be expressed as **A** = **VR**. Suppose that material parameters in the original space are homogeneous and isotropic, and they are expressed by scalar parameters ε_0 and μ_0. Considering the left Cauchy-Green deformation tensor, **B** = $\mathbf{V}^2$ = AA^T. The tensor **B** can be expressed in diagonal form $B = diag\left[\lambda_1^2, \lambda_2^2, \lambda_3^3\right]$. With i = 1, 2, 3 , λ_i are the eigenvalues of the tensor, or the principal stretches for an infinitesimal element.

Then, $\varepsilon' = \varepsilon_0 \mathbf{B}[\det(A)]^{-1}$ becomes $\varepsilon' = \varepsilon_0 \; diag \left[\frac{\lambda_1}{\lambda_2\lambda_3}, \frac{\lambda_2}{\lambda_3\lambda_1}, \frac{\lambda_3}{\lambda_1\lambda_2}\right]$ and

$$\mu' = \mu_0 \mathbf{B}[\det(A)]^{-1} \text{ becomes } \mu' = \mu_0 \; diag \left[\frac{\lambda_1}{\lambda_2\lambda_3}, \frac{\lambda_2}{\lambda_3\lambda_1}, \frac{\lambda_3}{\lambda_1\lambda_2}\right].$$

That shows the material parameters of the transformation material can be calculated from the principal stretches of the deformation induced by the space transformation. It should be noted that the transformed material parameter and the deformation tensor have the same principal directions. The rigid body rotation in deformations do not then contribute to the material parameters [7].

Assume that in a certain volume V of free space there exist electromagnetic fields $\mathbf{E}_0(\mathbf{r})$, $\mathbf{H}_0(\mathbf{r})$, created by sources located outside V. Work in the frequency domain, and these vectors are complex amplitudes of the fields. The main idea is to fill V with a material in such a way that after the volume is filled, the original fields $\mathbf{E}_0$ and $\mathbf{H}_0$ would be transformed to other fields. According to the design goals, MMs will perform the general linear field transformation defined as

$$\mathbf{E}(\mathbf{r}) = \mathrm{F}(\mathbf{r},\omega)\, \mathbf{E}_0(\mathbf{r}) + \sqrt{\frac{\mu_0}{\varepsilon_0}}\, \mathrm{A}(\mathbf{r},\omega)\, \mathbf{H}_0(\mathbf{r})$$

$$\mathbf{H}(\mathbf{r}) = \mathrm{G}(\mathbf{r},\omega)\, \mathbf{H}_0(\mathbf{r}) + \sqrt{\frac{\mu_0}{\varepsilon_0}}\, \mathrm{C}(\mathbf{r},\omega)\, \mathbf{E}_0(\mathbf{r})$$

$\mathrm{F}(\mathbf{r}, \omega)$, $\mathrm{G}(\mathbf{r}, \omega)$ $\mathrm{A}(\mathbf{r}, \omega)$ and $\mathrm{C}(\mathbf{r}, \omega)$ are arbitrary differentiable scalar functions.

Only the case of scalar coefficients are considered in the above relations, although the method can be extended to the most general linear relations between original and transformed fields by replacing scalar coefficients by arbitrary dyadics. Substituting those equations into the Maxwell equations and demanding that the original fields satisfy the free-space Maxwell equations, one finds that the transformed fields E and H satisfy Maxwell equations $\nabla\times \mathbf{E} = -j\omega\mathbf{B}$, $\nabla\times\mathbf{H} = j\omega\mathbf{D}$ in a medium with material relations for $\mathbf{B}$ and $\mathbf{D}$ that describe a bi-anisotropic medium, whose constitutive relations can written as

$$\mathbf{B} = \overline{\overline{\mu}} \bullet \mathbf{H} + \sqrt{\varepsilon_0\mu_0}\left(\overline{\overline{\chi}} + j\overline{\overline{\kappa}}\right)^T \bullet \mathbf{E} \quad \text{and}$$

$$\mathbf{D} = \overline{\overline{\varepsilon}} \bullet \mathbf{E} + \sqrt{\varepsilon_0 \mu_0}\left(\overline{\overline{\chi}} - j\overline{\overline{\kappa}}\right)^T \bullet \mathbf{H}$$

Comparing the equations derived, the required material parameters of the field-transforming medium are apparent as to the permittivity dyadic and the permeability dyadic [8].

For the 'Leonhardt Cloak', a geometry is characterized by the space-time metric and the matrix $g_{\alpha\beta}$. The metric tensor may vary as a function of the coordinates because space-time may be curved or because curved coordinates are used in flat space. The determinant g of $g_{\alpha\beta}$, measures the 4-D volume of an infinitesimal space-time element as the product of $\sqrt{-g}$ and the infinitesimal coordinate volume. Denote the matrix inverse of $g_{\alpha\beta}$ as $g^{\alpha\beta}$ where, as customary in GR, the position of the indices indicates that $g_{\alpha\beta}$ is covariant and $g^{\alpha\beta}$ is contravariant under coordinate transformations. As a starting point, use the result of GR that the free-space Maxwell equations can be written in the macroscopic form in Cartesian components,

$$\sum_i \frac{\partial D^i}{\partial x^i} = \rho \ , \qquad \sum_i \frac{\partial B^i}{\partial x^i} = 0 \ ,$$

$$\sum_i \vartheta^{ijk} \frac{\partial H_k}{\partial x^j} = \frac{\partial D^i}{\partial t} + j^i \, , \qquad \sum_{jk} \vartheta^{ijk} \frac{\partial E_k}{\partial x^j} = -\frac{\partial B^i}{\partial t}$$

where ϑ^{ijk} is the completely antisymmetric Levi-Civita tensor in Cartesian components and ϑ^{ijk} is 1 for all cyclic permutations of ϑ^{123}, −1 for all cyclic permutations of ϑ^{213} and zero otherwise. The spatial indices indicate that in this representation $\mathbf{E}_i$ and $\mathbf{H}_i$ form the components of vectors that are covariant under purely spatial transformations, whereas D_i and B_i constitute contravariant spatial vectors. The charge density ρ and the current density j are given by $\sqrt{-g}\, j^0$ and of the 4-current j^α. In empty but possibly curved space, the electromagnetic fields are connected by the constitutive equations in SI units,

$$\mathbf{D} = \varepsilon_0 \varepsilon \mathbf{E} + \mathbf{w}/c \times \mathbf{H} \quad \text{and} \quad \mathbf{B} = \mathbf{H}\frac{\mu}{\varepsilon_0 c^2} - \mathbf{w}/c \times \mathbf{E}.$$

In dielectric media, the **E**, **B** vectors generate electric polarizations and magnetizations that constitute the **D**, **H** fields. The constitutive equations are described here in implicit form, but one can also express them as **D**, **H,** as functions of **E** and **B**. In general, ε and μ are symmetric matrices because spacetime appears as an anisotropic medium, but in empty space, ε and μ are equal, as in perfect impedance matching. The w-vector describes a magneto-electric coupling between the magnetic and the electric fields. Some materials are magneto-electric, but the simplest example is a moving medium, because a moving medium responds to the electromagnetic field in locally comoving frames and Lorentz transformations from the laboratory frame naturally mix electric and magnetic fields. In explicit form,

$$\varepsilon = \mu = \frac{\sqrt{-g}}{g_{00}} g^{ij} \quad \text{and} \quad w_i = \frac{g_{0i}}{g_{00}} \ .$$

The ε and μ are constructed from the spatial components of the inverse metric tensor and from the determinant and the time-time component of the metric tensor, whereas the w-vector is given in terms of the time-space components of the metric tensor. Empty space appears as an impedance-matched anisotropic magneto-electric or moving medium. Since empty space appears as a medium, what happens if a medium appears as empty space? Suppose that the medium appears as the result of a coordinate transformation from some fictitious spacetime, say electromagnetic spacetime, to physical space. Electromagnetic spacetime could be flat with light traveling along straight lines, whereas for electromagnetic fields physical space appears to be curved, so bending light. Of course, this apparent curvature is the same type of illusion as straight lines appearing curved in curved coordinates, because in theory it is removable by the inverse coordinate transformation: but, in practice, one can exploit this apparent curvature to create illusions.

Using primes to denote the geometry and coordinates of electromagnetic spacetime and describing physical space in generalized coordinates x^i with spatial metric γ_{ij} and determinant γ, the theory is kept flexible. The metric γ_{ij} should differ from the effective geometry $g_{\alpha\beta}$ generated by the medium. The transformation rules of tensors in GR give rise to a simple recipe for the construction of media that facilitate spacetime coordinate transformations. Contravariant tensors are transformed as

$$g^{\alpha\beta} = \sum_{\alpha'\beta'} \frac{\partial x^{\alpha} \partial x^{\beta}}{\partial' x'^{\alpha'} \partial' x'^{\beta'}} \ g'^{\alpha'\beta'}$$

The transformed inverse metric serves as the building block of the dielectric tensors of the medium. If we wish to express physical space in generalized coordinates, consider a subtlety that appears when we write the divergences in Maxwell's equations in spatially covariant form

$$\frac{1}{\sqrt{\gamma}}\sum_i \frac{\partial\sqrt{\gamma}D^i}{\partial x^i} = \rho \qquad \text{and} \qquad \frac{1}{\sqrt{\gamma}}\sum_i \frac{\partial\sqrt{\gamma}B^i}{\partial x^i} = 0$$

The $\varepsilon^{\mathrm{ij}}$ and μ^{ij} tensors are naturally contravariant with respect to the background geometry of physical space, but γ differs from −g. Consequently, to write the medium as producing an active coordinate transformation, we need to multiply **D** and **B** by $\sqrt{\gamma}$ in the constitutive equations and re-scale ρ and **j** accordingly, which is also consistent with the form of the Levi-Civita tensor in curved coordinates in the curls in Maxwell's equations. However, a second subtlety arises from the curls. The coordinate transformation may turn a right-handed coordinate system into a locally left-handed one, but the curls implicitly assume a right-handed system, because ϑ^{ijk} changes sign under coordinate transformations from right to left-handed systems. Consequently, invert the sign of ε, μ and ρ, **j** wherever this is the case.

The transformation to a left-handed coordinate system thus corresponds to negative refraction. Taking all this into account, the simple recipe is

$$\varepsilon^{ij} = \mu^{ij} = \mp\frac{\sqrt{-g}}{\sqrt{\gamma}}\frac{g^{ij}}{g_{00}} \qquad \text{and} \qquad w_i = \frac{1}{\sqrt{\gamma}}\frac{g_{0i}}{g_{00}}$$

The ∓ sign indicates the handedness, minus for right-handed transformations and plus for locally left-handed ones. Starting from the inverse metric $g'^{\alpha\beta}$ in electromagnetic space, the $g^{\alpha\beta}$ matrix is calculated according to the transformation rule. The matrix inverse of $g^{\alpha\beta}$ gives the metric tensor $g_{\alpha\beta}$ and the inverse of the determinant of $g^{\alpha\beta}$ gives g. The equations specify the required electromagnetic properties that will turn the coordinate transformation into reality.

SECTION 3. CLOAKS

The ‘Huang Cloak’ sets out a construction using a concentric layered, homogeneous and isotropic structure that produces an anisotropic distribution of the permittivity required. For the sake of simplicity the problem is restricted to a two-dimensional case. Alternating layers, A and B, of different homogeneous isotropic dielectric materials are suggested. When the layers are sufficiently thin compared with wavelength, the whole layered structure can be treated as a single anisotropic medium with dielectric permittivity

$$\varepsilon_\theta = \frac{\varepsilon_A + \eta\varepsilon_B}{1+\eta} \qquad \text{and} \qquad \varepsilon_r = \frac{1}{1+\eta}\left(\frac{1}{\varepsilon_A} + \frac{\eta}{\varepsilon_B}\right)$$

where, ε_A, ε_B are the permittivities of the layer A and layer B, respectively, $\varepsilon_\theta, \varepsilon_r$ are respectively the angular and radial components of the effective anisotropic permittivity tensor and η is the thicknesses ratio of the two layers, $\eta = \frac{d_B}{d_A}$. Treating the layered structure as an effective anisotropic medium is based on the effective medium approximation that requires the thickness of each layer to be much less than the wavelength, and the number of the layers is large. When considering the finite thickness of a practical layered structure, it has been demonstrated in a planar case that the effective-medium approximation becomes more appropriate as the layers are made thinner.

Consider the electromagnetic wave scattering for an infinite conducting cylinder shelled either with a concentric layered structure or with an equivalent anisotropic medium. A plane wave with TM polarization is assumed to impinge along the x-direction upon the shelled cylinder. The axial components of the incident and scattered magnetic field vector **H** outside the cylinder may be expressed as

$$H_z^i = H_0 \sum_{n=-\infty}^{n=\infty} j^{-n} J_n(k_0 r) e^{jn\theta} \quad \text{and} \quad H_z^s = H_0 \sum_{n=-\infty}^{n=\infty} C_n j^{-n} H_n^{(2)}(k_0 r) e^{jn\theta}$$

where, J_n is the Bessel function and $H_n^{(2)}$ the Hankel function of the second kind. k_0 Is the wave number of the outside medium. The magnetic field inside the m^{th} dielectric layer (designated by the subscript m = 1 to 2N) of the layered structure is expressed by

$$H_{zm}^{t} = H_0 \sum_{n=-\infty}^{n=\infty} j^{-n} \left[A_{mn} J_n(k_m r) + B_{mn} H_n^{(2)}(k_m r) \right] e^{jn\theta}$$

where, k_m is the wave number in the m^{th} dielectric layer. In these equations, a time dependence of the form $e^{j\omega t}$ is assumed for the electromagnetic field quantities but is suppressed throughout. A_{mn}, B_{mn}, and C_n are arbitrary constants that can be determined by enforcing the boundary continuity condition at r = a, r = b, and at the interfaces between each dielectric layer. The electric fields in each medium are determined through

$$E_\theta = \frac{j\omega\mu}{k^2} \frac{\partial H_z}{\partial r} \qquad \text{and} \qquad E_r = -\frac{1}{r} \frac{j\omega\mu}{k^2} \frac{\partial H_z}{\partial \theta}$$

For a TM incident wave from a magnetic line source (with magnetic current I_m and located at $\mathbf{r} = \mathbf{r}_0$), the incident magnetic field is

$$H_z^i = -\frac{\omega \varepsilon_0 I_m}{4} H_0^{(2)}(k_0|\mathbf{r} - \mathbf{r}_0|)$$

The scattering electromagnetic fields can be analyzed similar to that of the plane wave case. When the conducting cylinder is shelled with the equivalent anisotropic medium, the electromagnetic fields scattering can be calculated similarly by application of Sommerfeld's bundle of rays field representation via. a polarization dependent coordinate transformation. The electromagnetic wave scattered by the cylinders can be verified by calculating the far-field scattering pattern that is proportional to

$$\xi(\theta) = \left| \sum_{n=-\infty}^{n=\infty} C_n e^{jn\theta} \right| .$$

Calculations show that as the layers are made thinner by increasing the number of layers from 10 to 40, the layered structure shell has nearly the same scattering property as the equivalent anisotropic medium shell. That means a concentric layered structure can be used to realize a cylindrical shell of a 2D rotationally invariant conducting cylinder shelled with a concentric layered structure of different number of layers, and that shelled with an anisotropic cylindrical medium. The thinner the layers, the better this layered structure approaches the scattering performance of the anisotropic medium.

Cloaking a central cylindrical region of radius ‘a’ by a concentric cylindrical shell of radius ‘b’ requires a cloaking shell with the following radius-dependent, anisotropic relative permittivity and permeability

$$\varepsilon_r = \mu_r = \frac{r-a}{r} \ , \quad \varepsilon_\theta = \mu_\theta = \frac{r}{r-a} \ , \quad \varepsilon_z = \mu_z = \left(\frac{b}{b-a}\right)\frac{r-a}{r}$$

Following TM wave illumination, only μ_z, ε_θ, and ε_r are of interest and must satisfy the requirements above. To make only one component spatially inhomogeneous and also to eliminate any infinite values, a reduced set of medium parameters can be chosen to completely remove the need for magnetic response of the material, that is especially important for making an optical frequency cloak. The shell with the reduced set of parameters provides the same wave trajectory inside the cloaking medium but will induce some unfavorable reflection at the outer boundary due to the impedance mismatch. Those reduced parameters are, [2]

$$\mu_z = 1 \ , \quad \varepsilon_\theta = \left(\frac{b}{b-a}\right)^2, \quad \varepsilon_r = \left(\frac{b}{b-a}\right)^2\left(\frac{r-a}{r}\right)^2.$$

SECTION 4. WHAT HAPPENS TO LIGHT?

Leonhardt [9] explains that according to Fermat’s Principle, light rays take the shortest optical paths in dielectric media and the refractive index, n, integrated along the ray trajectory defines the path length. When n is spatially varying the shortest optical paths are not straight lines and are usually curved. This light bending is the cause of many optical illusions, for example the desert mirage. Imagine a different situation where a medium guides light around a hole in it. Suppose that all parallel bundles of incident rays are bent around the hole and recombined in precisely the same direction as they entered the medium. An observer would not see the difference between light passing through the medium, propagating across empty space or, equivalently, in a uniform medium. Any object placed in the hole would be hidden from sight.

In order to carry images, light should propagate with a continuous range of spatial Fourier components, i.e. in a range of directions. Leonhardt says that the mathematical reason for the impossibility of perfect invisibility is the uniqueness of the inverse-scattering problem for waves. The scattering data, i.e. the directions and amplitudes of the transmitted plane-wave components, determine the spatial profile of the refractive index. Therefore, the scattering data of light in empty space are only consistent with propagation through empty space and perfect illusions are impossible due to the wave nature of light. On the other hand, this does not limit the imperfections of invisibility because they may be very small. Nor does it apply to light rays, i.e. to light propagation within the regime of geometrical optics. Leonhardt develops a general recipe for the design of media that create perfect invisibility for light rays over a continuous range of directions. Since the method is based on geometrical optics, the inevitable imperfections of invisibility can be made exponentially small for objects that are much larger than the wavelength of light.

Consider a dielectric medium that is uniform in one direction and light of wavenumber k that propagates orthogonal to that direction. The medium is characterized by the refractive-index profile n(x, y). In order to satisfy the validity condition of geometrical optics, n(x, y) must not significantly vary over the scale of an optical wavelength $\frac{2\pi}{k}$. To describe the spatial coordinates in the propagation plane, use complex numbers z = x + iy with the partial derivatives $\partial_x = \partial_z + \partial_z^*$ and $\partial_y = i\partial_z - i\partial_z^*$ where the star symbolizes complex conjugation. In the case of a gradually varying refractive-index profile both amplitudes of the two polarizations of light obey the Helmholtz equation

$$(4\partial_z^*\partial_z + n^2k^2)\varphi = 0$$

written here in complex notation with the Laplace operator $\partial_x^2 + \partial_y^2 = 4\partial_z^*\partial_z$. Now introduce new coordinates, w, described by an analytic function w(z) that does not depend on z*. Such functions define conformal maps that preserve the angles between the coordinate lines. Since

$$\partial_z^*\partial_z = \left|\frac{\partial w}{\partial z}\right|^2 \partial_w^*\partial_w \ ,$$

a Helmholtz equation is obtained in w-space with the transformed refractive-index profile n′ that is related to the original one by $n = n'\left|\frac{dw}{dz}\right|$. Suppose that the medium is designed such that n(z) is the modulus of an analytic function g(z). The integral of g(z) defines a map w(z) to new coordinates where, according that last equation, the transformed index n′ is unity. Consequently, in w-coordinates the wave propagation is indistinguishable from empty space where light rays propagate along straight lines. The medium performs an optical conformal mapping to empty space. If w(z) approaches z for w $\rightarrow\infty$, all incident waves appear at infinity as if they have traveled through empty space regardless of what has happened in the medium. However, as a consequence of the Riemann Mapping Theorem, nontrivial w-coordinates occupy Riemann sheets with several ∞, one on each sheet.

As an example, consider the mapping

$$w = z + \frac{a^2}{z}, \quad z = \frac{1}{2}\left(w \pm \sqrt{w^2 - 4a^2}\right)$$

that is realized by the refractive index profile $n = \left| 1 - \frac{a^2}{z^2} \right|$. The constant ‘a’ characterizes the spatial extension of the medium. The function above for ‘a’ and ‘z’ maps the exterior of a circle of radius ‘a’ on the z-plane onto one Riemann sheet and the interior onto another. Light rays traveling on the exterior w-sheet may pass the branch cut between the two branch points ±2a. In continuing their propagation, the rays approach ∞ on the interior w-sheet. Seen on the physical z-plane, they cross the circle of radius ‘a’ and approach the singularity of the refractive index at the origin. For general w(z), only one ∞ on the Riemann structure in w-space corresponds to the true ∞ of physical z-space and the others to singularities of w(z). Instead of traversing space, light rays may cross the branch cut to another Riemann sheet where they approach ∞. Seen in physical space, the rays are irresistibly attracted towards some singularities of the refractive index. Instead of becoming invisible, the medium casts a shadow that is as wide as the apparent size of the branch cut is wide. All that needs to be done is to guide light back from the interior to the exterior sheet, i.e., seen in physical space, from the exterior to the interior layer of the device. To find the required refractive index profile, interpret the Helmholtz equation in w-space as the Schrödinger equation of a quantum particle of effective mass k^2 moving in the potential U with energy E such that $U - E = -n'^2/2$.

To send all rays that have passed through the branch cut onto the interior sheet back to the cut at precisely the same location and in the same direction they entered, a potential is required for which all trajectories are closed. Assuming radial symmetry for U(w) around one branch point w_1, say +2a in the example, only two potentials have this property, the harmonic oscillator and the Kepler potential. In both cases the trajectories are ellipses that are related to each other by a transmutation of force according to the Arnol’d-Kasner theorem. The harmonic oscillator corresponds to the transformed refractive-index profile n′ with

$$n'^2 = 1 - \frac{|w - w_1|^2}{r^2}$$

where r is a constant radius. The Kepler potential with negative energy E is realized by the profile with

$$n'^2 = \frac{r}{|w - w_1|} - 1$$

Note that the singularity of the Kepler profile in w-space is compensated by the zero of |dw/dz| at a branch point in physical space such that the total refractive index is never singular. In both the above cases, r defines the radius of the circle on the interior w-sheet beyond which n'^2 would be negative and hence inaccessible to light propagation. This circle should be large enough to cover the branch cut. The inverse map z(w) turns the outside of the circle into the inside of a region bounded by the image z(w) of the circle line in w-space. No light can enter this region and everything inside is invisible.

Light is refracted at the boundary between the exterior and the interior layer. Seen in w-space, light rays encounter a transition from the refractive index 1 to n′. Fortunately, refraction is reversible. After the cycles on the interior sheets light rays are refracted back to their original directions. Invisibility will not be affected unless the rays are totally reflected.

According to Snell’s Law, rays with angles of incidence θ with respect to the branch cut enter the lower sheet with angles θ' such that $n' \sin \theta' = \sin\theta$. If $n' < 1$ this equation may not have real solutions for θ larger than a critical angle, Θ. Instead of entering the interior layer of the device, the light is totally reflected. The angle Θ defines the acceptance angle of the dielectric invisibility device, because beyond Θ the device appears silvery instead of invisible. The transformed refractive-index profiles at the boundary between the layers are lowest at the other branch point w_2 that limits the branch cut, being $w_2 = -2a$ in the example.

In the case of the harmonic oscillator profile n′ lies always below 1 and the acceptance angle is

$$\Theta = \arccos\left(\frac{|w_2 - w_1|}{r}\right)$$

For all-round invisibility, the radius r should approach infinity, which implies that the entire interior sheet is employed for guiding the light back to the exterior layer. Fortunately, the Kepler profile, $n'^2 = \frac{r}{|w - w_1|} - 1$, does not lead to total reflection if $r \geq 2|w_2 - w_1|$. In this case, the invisible area is largest for $r = 2|w_2 - w_1|$.

SECTION 5. SHAPING THE CLOAK

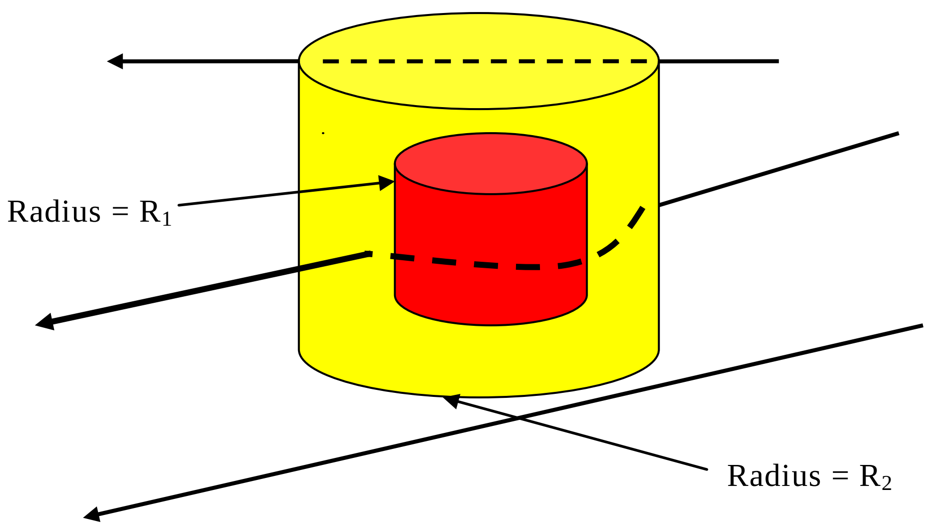
The 'Leonhardt Cloak'. Perfect invisibility devices facilitate coordinate transformations with holes in physical space. In this way, electromagnetic radiation is naturally guided around such excluded regions and the device requires anisotropic media, because the inverse scattering problem for waves in isotropic media has unique solutions. They also require media where the phase velocity of light approaches infinity at the inside of the cloaking layer, because coordinate transformations with holes rip apart points of zero volume in electromagnetic space and tear them to finite volumes in physical space, unless the coordinate transformation is singular. To prove this, consider the determinant of the ε and μ tensor. For an invisibility device only space is transformed, so w vanishes, g_{00} is unity and $-g$ is reduced to the inverse of the determinant of the spatial components g_{ij} . In 3D space in terms of the Jacobian, J

$$\det \varepsilon = \sqrt{-g}\ \ \gamma^{-\frac{3}{2}} = \sqrt{(-g')}\ \ \gamma^{-\frac{3}{2}} J \quad , \qquad J = \frac{\partial(x'^1, x'^2, x'^3)}{\partial(x^1, x^2, x^3)} .$$

At a point of measure zero, $\sqrt{(-g')}$ vanishes and so does det ε, the product of the eigenvalues of ε and μ. Consequently, at least one of the eigenvalues of ε and μ are zero. Therefore in at least one direction of the anisotropic medium the phase-velocity of light diverges near the inside of the cloak.

The speed of light in media can reach high values in narrow frequency ranges or, naturally, for static fields. On the other hand, invisibility devices that are only perfect within the limits of geometrical optics are not subject to such constraints.

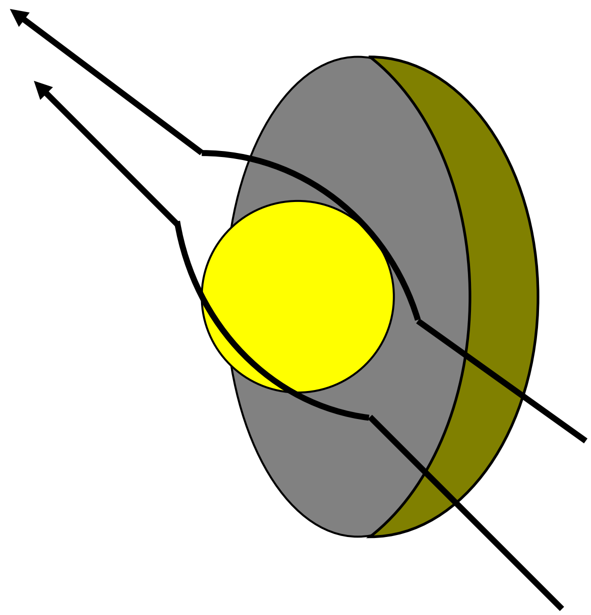
A cylindrical invisibility device that stretches the z-axis out in the radial direction to a full cylinder of radius R_1, compressed within a cylindrical volume of radius R_2, is an invisibility cloak of thickness $R_2 - R_1$ where anything placed inside the inner radius R_1 is hidden. In the drawing below, the cloaking device has been drawn 3-D and the path of light rays is shown.



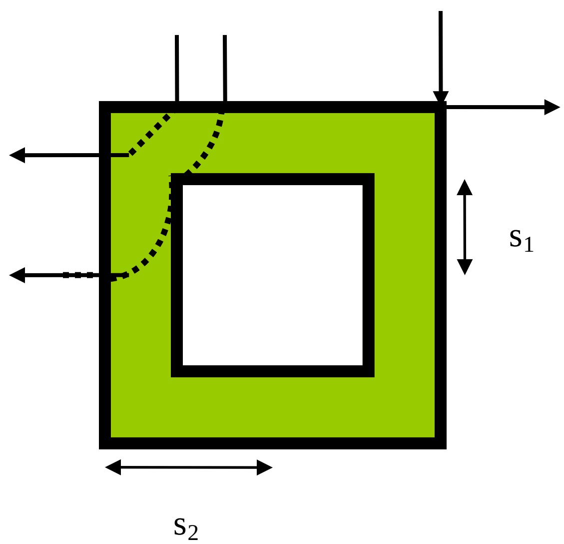
Then,

$$r = R_1 + r' \frac{R_2 - R_1}{R_2} \Rightarrow \varepsilon^i_j = \frac{R_2}{R_2 - R_1} \frac{r'}{r}\ diag\left(\frac{(R_2 - R_1)^2}{R_2^2}, \frac{r^2}{r'^2}, 1\right)$$

within $R_1 \le r \le R_2$ or, equivalently, $0 \le r' \le R_2$. Clearly, close to the lining of the cloak at $r \to R_1$ where $r' \to 0$, the speed of light in the r and z directions diverges, whereas in φ-direction the phase velocity tends to zero. The most logical shape is spherical and a spherical cloak would conform to the basic requirements concerning the deviation of the light path in the cut-away diagram below.



The 'Pendry Square Cloak' described in [6] uses coordinate transformation equations for the electromagnetic design of a square-shaped cloak with a side length $2s_1$ of the inner square and a side length $2s_2$ of the outer square as shown below with the transformation equations operating within the green region enclosed between the two squares. The path of light rays is shown.



Those equations are

$$x_1'(x_1, x_2, x_3) = \frac{s_2 - s_1}{s_2} + s_1 \ ,$$

$$x_2'(x_1,x_2,x_3) = x_2\left(\frac{s_2 - s_1}{s_2} + \frac{s_1}{x_1}\right) \quad ,$$

$$x_3'(x_1,x_2,x_3) = x_3 .$$

with the Jacobi matrix and its determinant

$$A_i^{i'} = \begin{pmatrix} \frac{s_2 - s_1}{s_2} & 0 & 0 \\ -\frac{x_2}{x_1^2} s_1 & \frac{s_2 - s_1}{s_2} + \frac{s_1}{x_1} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad ,$$

$$\det(A_i^{i'}) = \frac{s_2 - s_1}{s_2}\left(\frac{s_2 - s_1}{s_2} + \frac{s_1}{x_1}\right)$$

for ($0 < x_1 \leq s_2$), ($-s_2 < x_2 \leq s_2$), $|x_2| < |x_1|$ and ($|x_3| < \infty$), with the transformation continuous at the boundary of the transformed domain. The corresponding transformation formulas for other domains of the square cloak can be obtained by applying rotation operators with rotation angles $\pi/2$, π and $3\pi/2$ around the z-axis to the coordinate transformation equations. The transformation expands the space within the inner square at the expense of a compression of space between the inner and outer square. Because of the relationships between $\det(A_i^{i'})$ and ε and μ, the relative permittivity and the relative permeability tensors can be found expressed in the coordinates of the transformed space.

Due to the natural invariance of the Minkowski equations, the permittivity and permeability tensors can also be interpreted as the material properties of a medium described in the coordinate system of the original space by substituting the primed indices by unprimed indices. The relative permittivity and permeability tensors are non-diagonal, which is a direct consequence of the non-conformality of the transformation. However, in terms of fabricating such a material, it is desirable to have the material parameters denoted in their eigenbasis. Here, the permittivity and permeability tensors are diagonal. Due to the symmetry of the tensors, an eigenbasis solution always exists.

Unfolded geometry may provide a means to extend the shape possibilities of a cloaking device. Solutions for the fields in a coated cylinder where the core radius is bigger than the shell radius are seemingly unphysical, but can be given a physical meaning if one transforms to an equivalent problem by unfolding the geometry. In particular the unfolded material can act as an impedance matched hyperlens, and as the loss in the lens goes to zero, finite collections of polarizable line dipoles laying within a critical region surrounding the hyperlens are shown to be cloaked having vanishingly small dipole moments. This cloaking, which occurs both in the folded geometry and the equivalent unfolded one, is due to anomalous resonance, where the collection of dipoles generates an anomalously resonant field. That acts back on the dipoles to essentially cancel the external fields acting on them [10].

SECTION 6. SUGGESTION

In the Introduction the Philadelphia Experiment was mentioned. We devote this last section to a proposal as to how something as large and three-dimensional as the U.S.S. Eldridge would be rendered invisible to sight, infra-red and radar detection using the principles above set out. As the ship is taken to have an arbitrary shape, reference [60] will be followed.

Express Maxwell's equations in a Cartesian (x, y, z) space to an arbitrary curved space (q_1, q_2, q_3). Then,

$$x = f_1(q_1, q_2, q_3),$$

$$y = f_2(q_1, q_2, q_3),$$

$$z = f_3(q_1, q_2, q_3).$$

The length of a line element in the transformed space is given by

$$dl^2 = [dq_1, dq_2, dq_3]gg^T[dq_1, dq_2, dq_3]^T ,$$

where the superscript 'T ' denotes the transpose of matrix

$$g = \begin{pmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_2}{\partial q_2} & \frac{\partial f_3}{\partial q_3} \\ \\ \frac{\partial f_1}{\partial q_1} & \frac{\partial f_2}{\partial q_2} & \frac{\partial f_3}{\partial q_3} \\ \\ \frac{\partial f_1}{\partial q_1} & \frac{\partial f_2}{\partial q_2} & \frac{\partial f_3}{\partial q_3} \\ \end{pmatrix}$$

The volume of a space element is

$$dv = \det(g)\ dq_1,\ dq_2,\ dq_3.$$

and the spacetime metric tensor is

$$g_{\alpha\beta} = \text{diag}\ [1, (-gg^T)]$$

Maxwell's equations in Cartesian coordinates become, in curved coordinates (with stars meaning those quantities in curved coordinates)

$$\nabla \bullet \mathbf{B} = 0 \quad \rightarrow \quad \nabla_q \bullet \mathbf{B^*} = 0\ ,$$

$$\nabla \bullet \mathbf{D} = \rho \quad \rightarrow \quad \nabla_q \bullet \mathbf{D^*} = \rho^*\ ,$$

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B} \quad \rightarrow \quad \nabla_q \times \mathbf{E^*} - \frac{\partial}{\partial t}\mathbf{B^*}\ ,$$

$$\nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial}{\partial t}\mathbf{D} \quad \rightarrow \quad \nabla_q \times \mathbf{H^*} = \mathbf{j^*} + \frac{\partial}{\partial t}\mathbf{D^*}\ ,$$

$$\mathbf{D} = \varepsilon_0 \overline{\overline{\varepsilon}} \bullet \mathbf{E} \quad \rightarrow \quad \mathbf{D^*} = \varepsilon_0 \overline{\overline{\varepsilon^*}} \bullet \mathbf{E^*} \quad = \varepsilon_0 \left[\det(g)(g^T)^{-1} \overline{\overline{\varepsilon}}\ g^{-1}\right] \bullet \mathbf{E^*}\ ,$$

$$\mathbf{B} = \mu_0 \overline{\overline{\mu}} \bullet \mathbf{H} \quad \rightarrow \quad \mathbf{B^*} = \mu_0 \overline{\overline{\mu^*}} \bullet \mathbf{H^*} = \mu_0 \left[\det(g)(g^T)^{-1} \overline{\overline{\mu}}\ g^{-1}\right] \bullet \mathbf{H^*},$$

$$\mathbf{j^*} = \mathbf{j}\ \left[\det(g)(g^T)^{-1}\right] \quad \text{and} \quad \rho^* = \rho\left[\det(g)\right]\ .$$

The inverse of the matrix is denoted by the index '-1'. Therefore,

$$\mathbf{E}^* = g\mathbf{E} \quad \text{and} \quad \mathbf{H}^* = g\mathbf{H}$$

The cloak should compress an enclosed space with the exterior boundary without changing it. That is represented by the coordinate transformation $x = f_1(q_1, q_2, q_3)$, $y = f_2(q_1, q_2, q_3)$, $z = f_3(q_1, q_2, q_3)$, connecting points in the compressed space with coordinates (q_1, q_2, q_3) and points (x, y, z) in the original space. The exterior boundary satisfies $q_1 = x$, $q_2 = y$ and $q_3 = z$.

Since the eigenfunctions of the wave equations in the cloak medium are given by $\mathbf{E}^* = g\mathbf{E}$ and $\mathbf{H}^* = g\mathbf{H}$, these easily relate to the eigenfunctions of the wave equations in the uncompressed space. The cloak must be able to exclude light from the ship without perturbing the exterior field and should be without reflections. For the cloaking, the cloak is located in air, so simplifying permittivity and permeability from $\det(g)(g^T)^{-1}\overline{\overline{\varepsilon}}\, g^{-1}$ and $\det(g)(g^T)^{-1}\overline{\overline{\mu}}\, g^{-1}$ to $\overline{\overline{\varepsilon *}} = \overline{\overline{\mu *}}$.

If the transmitted electric field $\mathbf{E^*}^i$ and transmitted magnetic field $\mathbf{H^*}^i$ are not to interact with the inner boundary, for external incident waves having $\mathbf{E}^i$ and $\mathbf{H}^i$,

$$\mathbf{E}^{i} * = g\mathbf{E}^{i} \quad \text{and} \quad \mathbf{H}^{i} * = g\mathbf{H}^{i},$$

and these fields satisfy Maxwell's equations in the cloak medium. If the tangential components of $\mathbf{E}^i$ and $\mathbf{H}^i$, and, $\mathbf{E}^i*$ and $\mathbf{H}^i*$ remain continuous along the cloak, there will be no reflections.

Let the subscript 'n' represent the normal direction pointing outward from the cloak and the 't' subscripts be two tangential directions vertical with each from the cloak and decompose as follows.

$$\mathbf{E}^{i} * = \left[E *^{i}_{n},\ E *^{i}_{t_1},\ E *^{i}_{t_2}\right] \quad \text{and} \quad \mathbf{H}^{i} * = \left[H *^{i}_{n},\ H *^{i}_{t_1},\ H *^{i}_{t_2}\right]$$

If $\left[\ n^*,\ t_1^*,\ t_2 *\right]$ are the orthogonal to each other unit vectors in those directions,

$$\mathbf{E}^{i} * = g\mathbf{E}^{i} \quad \Rightarrow \quad \begin{bmatrix} E *^{i}_{n} \\ E *^{i}_{t_1} \\ E *^{i}_{t_2} \end{bmatrix} = \left[\ n^*,\ t_1^*,\ t_2 *\right]^{-1} g\ \mathbf{E}^{i}\ ,$$

$$\mathbf{H}^{i}* = g\mathbf{H}^{i} \quad \Rightarrow \quad \begin{bmatrix} H*_{n}^{i} \\ H*_{t_1}^{i} \\ H*_{t_2}^{i} \end{bmatrix} = \begin{bmatrix} n*, & t_1*, & t_2* \end{bmatrix}^{-1} g \ \mathbf{H}^{i} \ .$$

At the exterior boundary of the cloak, because $q_1 = x$, $q_2 = y$ and $q_3 = z$,

$$f_1(q_1, q_2, q_3) - q_i = 0 \ , \ i = 1, 2, 3 \ ,$$

and that characterizes the exterior boundary. If $C_1* = x*$, $C_2* = y*$, $C_3* = z*$, the vectors $\nabla_q f_i - C_i*$ (i = 1, 2, 3) lay in the same line as the normal direction, n, of the outer edge of the cloak. When $\nabla_q f_i - C_i* = 0$ the vector in the n-direction has zero magnitude, that is, **0**n*. Then, g on the outer cloak surface is

$$g = \left[F_1 n* + x*, \ F_2 n* + y*, \ F_3 n* + z*\right]$$

with

$$|F_i| = \left[\left(\frac{\partial f_i}{\partial q_i} - 1\right)^2 + \left(\frac{\partial f_i}{\partial q_j}\right)^2 + \left(\frac{\partial f_i}{\partial q_k}\right)^2\right]^{\frac{1}{2}},$$

$$i,j,k, = 1, 2, 3. \qquad i \neq j \neq k.$$

When the $\nabla_q f_i - C_i*$ direction points opposite to the n-direction, $F_i = -|F_i|$. When the $\nabla_q f_i - C_i*$ direction is the same as the n-direction, $F_i = |F_i|$.

Substituting $g = \left[F_1 n* + x*, \ F_2 n* + y*, \ F_3 n* + z*\right]$ into the previous decomposed equations,

$$E_{t_1}^{i} = \mathbf{E}^{i} \bullet t_1*, \quad E_{t_2}^{i} = \mathbf{E}^{i} \bullet t_2*, \quad H_{t_1}^{i} = \mathbf{H}^{i} \bullet t_1*, \quad H_{t_2}^{i} = \mathbf{H}^{i} \bullet t_2*$$

indicating that the tangential components are continuous and therefore, there will be no reflections at the exterior boundary.

Only when g is a symmetrical matrix will the exterior boundary be a perfectly matched layer where the permittivity and permeability have the form $\overline{\overline{\varepsilon*}} = \overline{\overline{\mu*}}$ with the principle axes in the n, t_1 and t_2 directions.

The equation $g = \lfloor F_1 n^* + x^*,\ F_2 n^* + y^*,\ F_3 n^* + z^* \rfloor$ shows that $g^T t_1^* = t_1^*$ and $g^T t_2^* = t_2^*$. Therefore, t_1^* and t_2^* are the eigenvectors of g^T with the same eigenvalue = 1. Considering now that g is a symmetrical matrix, the other eigenvector of g^T is n* with eigenvalue det(g). Thus, n*, t_1 and t_2 are eigenvectors of eigenvalues

$$\left(gg^T\right)^{-1} = (g^T)^{-1} g^{-1} \rightarrow \frac{1}{\det(g)^2},\ 1\ ,\ 1$$

The symmetric g now dictates that

$$\overline{\overline{\varepsilon^*}} = \overline{\overline{\mu^*}} = diag\ \left[\frac{1}{\det(g)},\ \det(g),\ \det(g)\right]$$

with the diagonal elements respectively corresponding to the three principal axes already discussed. As to line and point transformation at the inner cloak boundary, it is necessary to show that no reflections occur and no field can penetrate into the cloaked region.

Considering **line transformed cloaks**, assume that x = b_1(s) , y = b_2(s) , z = b_3(s) characterize the line mapped to the inner boundary. Then

$$f_1(q_1, q_2, q_3) = b_1(s),$$

$$f_2(q_1, q_2, q_3) = b_2(s),$$

$$f_3(q_1, q_2, q_3) = b_3(s)$$

and each point (b_1, b_2, b_3) on the line maps to a closed curve on the inner boundary. The parameter s can be expressed as a function of (q_1, q_2, q_3) with

$$s = u(q_1, q_2, q_3)$$

so that

$$\nabla_q s = \left(\frac{\partial u}{\partial q_1} x^* + \frac{\partial u}{\partial q_2} y^* + \frac{\partial u}{\partial q_3} z^*\right)$$

is the gradient of s pointing in the direction of the greatest rate of increasing s. Further,

$$\nabla_q b_i = \frac{\partial b_i}{\partial s} \nabla_q s \ , \quad \text{i = 1, 2, 3}$$

meaning that $\nabla_q s$ and $\nabla_q b_i$ both have the same direction. Decompose the incident fields at the inner boundary using 'n' to mean the normal direction at the inner boundary pointing outwards from the cloaked region and the 't' denotes the tangential directions of the inner boundary with t_1 vertical with the plane determined by two vectors in n and $\nabla_q s$ directions. The other 't' is t_2 and that is vertical with t_1. Since s varies on the inner surface, the direction of $\nabla_q s$ denoted by s_q should not be parallel to the inner surface normal direction. Thus, the plane determined by the vectors in n and $\nabla_q s$ directions always exists. The n, t_1, and t_2 directions are unique because a line being cloaked can be curved rather than straight, whereas a point is otherwise.

Since at the inner boundary $f_1(q_1, q_2, z) - b_i(s) = 0$ (i = 1, 2, 3), $\nabla_q f_i - \nabla_q b_i$ is the characteristic normal direction of the inner boundary. There, g can be written as

$$g = \lfloor F_1 n * + \nabla_q b_1 \ , \ F_2 n * + \nabla_q b_2 \ , \ F_3 n * + \nabla_q b_3 \rfloor \ .$$

Here,

$$|F_i| = \left[\left(\frac{\partial f_i}{\partial q_1} - \frac{\partial b_i}{\partial q_1} \right)^2 + \left(\frac{\partial f_i}{\partial q_2} - \frac{\partial b_i}{\partial q_2} \right)^2 + \left(\frac{\partial f_i}{\partial q_3} - \frac{\partial b_i}{\partial q_3} \right)^2 \right]^{\frac{1}{2}}$$

Again, when $\nabla_q f_i - \nabla_q b_i$ is in the same direction as n, $|F_i| = F_i$ but if in the opposite direction, $F_i = -|F_i|$. With n and s_q, t_1 is orthogonal. If s* denotes the unit vector in the direction of $\nabla_q s$, substituting the equation for g into the previous 'decomposed' equations, $E_{t_1}^i * = H_{t_1}^i * = 0$, but the other fields are not zero.

To find the permittivity and permeability at the inner boundary, use $g = \lfloor F_1 n * + \nabla_q b_1 \ , \ F_2 n * + \nabla_q b_2 \ , \ F_3 n * + \nabla_q b_3 \rfloor$. It is not difficult to show that $g^T t_1 * = 0$ indicating that $g g^T t_1 * = 0$. One of the eigenvectors is $t_1 *$ and the eigenvalue is $\lambda_{t_1} = 0$. That implies det(g) = 0. Because $g g^T$ is a symmetrical matrix, the other eigenvectors orthogonal to each other in the $(n - t_2)$ plane are denoted a* and b* with eigenvalues λ_a and λ_b with

$$\lambda_a \lambda_b = \left| n^* \times \ s^* \right|^2 \ \left| (F_1 x^* + F_2 y^* + F_3 z^*) \times (B_1 x^* + B_2 y^* + B_3 z^*) \right|^2 .$$

Now,

$$\lambda_{t_1} \lambda_a \lambda_b = \det(gg^T) = \det(g)^2,$$

then

$$\lambda_{t_1} = \frac{\det(g)^2}{\lambda_a \lambda_b} .$$

Because

$$\overline{\overline{\varepsilon^*}} = \overline{\overline{\mu^*}} = \det(g)(g^T)^{-1} g^{-1}$$

then

$$\overline{\overline{\varepsilon^*}}(gg^T) = \overline{\overline{\mu^*}}(gg^T) = \det(g) .$$

So $t_1{}^*$, a* and b* are not only the principal axes at the inner boundary, they correspond with the diagonal elements of

$$\overline{\overline{\varepsilon^*}} = \overline{\overline{\mu^*}} = diag \left[\frac{\lambda_a \lambda_b}{\det(g)}, \ \frac{\det(g)}{\lambda_a}, \ \frac{\det(g)}{\lambda_b} \right] \text{ with det(g) = 0.}$$

So $\varepsilon_a = \mu_a = \varepsilon_b = \mu_b = 0$ and that says the inner boundary is isotropic in the (n − t_2) plane and n* and t_2* are principal axes with $\varepsilon_n = \mu_n = \varepsilon_{t_2} = \mu_{t2} = 0$ and the inner boundary operates as a perfect electric conductor/perfect magnetic conductor combination. That will support electric and magnetic surface displacement currents in the t_1 direction and for zero reflection, the boundary conditions at this PEC/ PMC combined layer requires that the incident electric (magnetic) fields in t_1 direction and normal electric (magnetic) displacement fields are zero.

Using $E_{t_1}^i{}^* = H_{t_1}^i{}^* = 0$ and because $\varepsilon_n = \mu_n = 0$,

$$D_n^i{}^* = B_n^i{}^* = 0 .$$

Therefore, there will be no reflections from the inner cloak surface and the PEC/PMC combination will allow no field to penetrate the cloaked volume. In the t_1 direction, induced displacement surface currents make $E_{t_2}^i$ * and $H_{t_2}^i$ * go to zero but are discontinuous across the inner boundary.

For **point transformed cloaks**, a point with the coordinate [c_1, c_2, c_3] maps to the inner boundary.

$$c_1 = f_1(q_1, q_2, q_3),$$

$$c_2 = f_2(q_1, q_2, q_3),$$

$$c_3 = f_3(q_1, q_2, q_3);$$

$$\mathbf{E}^{i}{*} = \left\lfloor E{*}^{i}_{n},\ E{*}^{i}_{t_1},\ E{*}^{i}_{t_2} \right\rfloor \quad \text{and} \quad \mathbf{H}^{i}{*} = \left\lfloor H{*}^{i}_{n},\ H{*}^{i}_{t_1},\ H{*}^{i}_{t_2} \right\rfloor;$$

$$g = diag\ \left\lfloor F_1 n^*,\ F_2 n^*\ ,\ F_3 n^* \right\rfloor;$$

$$|F_i| = \left[\left(\frac{\partial f_i}{\partial q_1} \right)^2 + \left(\frac{\partial f_i}{\partial q_2} \right)^2 + \left(\frac{\partial f_i}{\partial q_3} \right)^2 \right]^{\frac{1}{2}} \qquad i = 1, 2, 3$$

$$E^{i}_{t_1}{*} = H^{i}_{t_1}{*} = 0\ ;\ E^{i}_{t2_1}{*} = H^{i}_{t_2}{*} = 0$$

$$E^{i}_{n}{*} = \left[F_1,\ F_2,\ F_3 \right] \mathbf{E}^{\mathrm{i}} \quad ; \quad H^{i}_{n}{*} = \left[F_1,\ F_2,\ F_3 \right] \mathbf{H}^{\mathrm{i}}\ .$$

The subscript 'n' the normal direction directed outwards from the cloaked region with t_1 and t_2 representing two tangential directions that are vertical with each other. Unlike the line case, tangential fields are all zero, implying that no field discontinuity exists at the inner boundary. Again, n* is eigenvector of gg^T with the eigenvalue $\lambda_n = F_1^2 + F_2^2 + F_3^2$ and the other eigenvectors are t_1*and t_2 * with the corresponding eigen values $\lambda t_1 = \lambda t_2 = 0$, indicating det(g) = 0. Carrying on in like manner with $\lambda_n \lambda t_1 \lambda t_2 = \det(g)^2$ and substituting for λ_n, the result is $\varepsilon_n = \mu_n = 0$.

Since tangential components of incident fields at the inner boundary are zero, there can be no reflections there and no external field will penetrate into the cloaked region. Therefore, the invisibility of invisibility cloaks with arbitrary shape constructed by general coordinate transformations is confirmed.

In essence, the basics are simple, but at the present time, the practicalities are a little more difficult to realize. Probably the most difficult ship to deal with would be a Second World War destroyer-shaped vessel. That had masts, antennae wires, rotating radar horn, guns and things hanging from or protruding from it. On the other hand, an aircraft carrier might be easier on account of its large flat surfaces and relatively small superstructure. Practically, one needs to redesign the ship (now called ‘Eldridge’) to incorporate a smooth, curved, aerodynamic-looking superstructure.

If fields cannot enter the cloak, fields generated within the cloaked region can be either reflected and/or scattered from or absorbed by the interior of the PEC layer. Would that be a problem? In peacetime, one would not wish to employ the cloak and so gratuitously ‘give away’ its presence. Under combat conditions, one would not wish to transmit signals for fear of being triangulated. Orders to be received by radio and updates to be transmitted would require the cloak to be disengaged. In the papers considered, we have no recollection of any worker saying how to ‘pull the plug’ on a cloaking device to make the cloak inoperative. Later, that will require a little discussion concerning whether it is necessary.

If a cloaked Eldridge were to be built now, one would not wish to take advantage of present stealth technology in that certain shapes and coatings can impart to a structure a small radar ‘footprint’. Further, the World War II-shaped ship would not be contemplated but instead, a stealth ship with cloaking capability where everything is concealed inside the smooth MM covered superstructure. That could well have discontinuities (holes) for guns to discharge. If the guns were WW II battleship sized (16 inch shell) or say, one metre diameter (for missile firing), those would be optically visible from only close proximity.

The question remains, how does one navigate a cloaked ship in pitch blackness where the only radiation that can enter the cloaked volume is, presumably, gravity – or, from illumination inside the PEC layer? We may need at least one window or, the device needs to be turned off to check navigational position – or – a small uncloaked observation platform.

The hull of our Eldridge could be of composite structure of an exterior MM forming the cloak. An air or air exhausted space is envisaged between the cloak and the interior PEC lining. Anything within that lining, the steel hull of the ship, would be invisible. With a smooth design, the difficulties associated with a conventional-looking ship disappear.

Another aspect is the crew. No discussion will be entertained here concerning the physiological or mental effects of the cloak on crew members, if any. In any case, if the reports of what happened to the original and subsequent crews of the U.S.S. Eldridge are to be believed, any such effects using an MM cloak cannot be worse than those reported in the early 1940s.

Whilst we mention a conventional steel hull, there is the possibility that the hull and all parts of the ship superstructure could be constructed from a synthetic material mechanically stronger than steel (carbon fibre-like composite?), containing the required ingredients to ensure that the entire hull acts as an MM with engineered properties. That suggestion lies within the province of those with extensive knowledge of materials. Let us address the problem of the superstructure and a steel hull exterior to the MM/PEC layers. If the cloaking effect could be spherically or cylindrically extended to include a volume greater than that occupied by the entire ship, that problem would be resolved. As to that, [34] provides a method. To extend the cloak requires an 'anti-object'. The idea is based on combining the concept of complementary media and transformation optics. The 'anti-object' is embedded in a negative index shell and can make an object that lies outside the cloaking shell invisible. The key idea behind this design is the complementary media that can be regarded as a special kind of transformation media. According to coordinate transformation theory, when a space is transformed into another space of different shape and size, the permittivity ε' and permeability μ' in the transformed space x' are given by

$$\varepsilon' = \mathbf{A}\,\varepsilon\,\mathbf{A}^{T}(\det\mathbf{A})^{-1} \quad \text{and} \quad \mu' = \mathbf{A}\,\mu\,\mathbf{A}^{T}(\det\mathbf{A})^{-1}$$

where **A** is the Jacobian transformation tensor.

The transformation media exhibits two properties. The optical path in the transformation media is exactly the same as that in the original space. The transformation media is reflectionless as long as the outer boundary coordinates before and after a coordinate transformation remain unchanged. Based on these principles, a kind of complementary media can be designed under a special kind of coordinate transformation, i.e. folding a piece of space into another. When a wave crosses the folding line from the original space into the folded space, it starts to experience a negative optical path as that in the original space.

The complementary media can be obtained by a coordinate transformation of folding the layer of air ($b < r < c$) into the layer of complementary media ($a < r' < b$). Here a , b and c are the core radius, the outer radius of complementary layer, and the outer radius of the canceled air layer, respectively. Consider a general coordinate transformation of the form $r = f(r')$, in which $f(r')$ is a continuous function of r' that satisfies $f(b) = b$ and $f(a) = c$. The ε' and μ' of the complementary layer can be written as

$$\varepsilon_r' = \mu_r' = \frac{f(r')}{r'}\frac{1}{f'(r')}$$

$$\varepsilon_\theta' = \mu_\theta' = \frac{r'}{f(r')}f'(r')$$

$$\varepsilon_z' = \mu_z' = \frac{f(r')}{r'}f'(r')$$

For a simple linear function, ε' and μ' are anisotropic. For the specific choice of $f(r') = \frac{b^2}{r^2}$ and $c = \frac{b^2}{a}$, we obtain isotropic parameters ε' and μ' . For TE waves,

$$\mu_r' = \mu_\theta' = -1.$$

For TM waves,

$$\varepsilon_z' = -\frac{b^4}{r'^4}.$$

The radial and tangential components are constants. If we let the core material be a perfect electric conductor (PEC), the system becomes the so-called superscatterer with an effective PEC boundary at radius c. Using a transformation that compresses a large circle of air with radius c into a small circle with radius 'a',

for TE waves, $$\mu_r'' = \mu_\theta'' = \varepsilon_z''\frac{a^2}{c^2} = 1 .$$

for TM waves, $$\varepsilon_r'' = \varepsilon_\theta'' = \mu_z''\frac{a^2}{c^2} = 1 .$$

With such a choice of dielectric core material, the wave experiences the same optical path as that in a circle of air with a radius 'c'. In that way, the whole system, including the outer air layer, the complementary media layer and the core material, is optically equal to a circle of air with radius 'c', and all are invisible to any form of external illumination.

Now suppose an object of permittivity ε_0 and permeability μ_0 is added in the outer circular layer of air that needs to be invisible. According to transformation media theory, it is always possible to include a complementary 'image' object with parameters ε'_0 and μ_0 in the complementary media layer

$$\frac{\varepsilon_0'_r}{\varepsilon_{0r}} = \frac{\mu_0'_r}{\mu_{0r}} = \frac{f(r')}{r'} \frac{1}{f'(r')}$$

$$\frac{\varepsilon_0'_\theta}{\varepsilon_{0\theta}} = \frac{\mu_0'_\theta}{\mu_{0\theta}} = \frac{r'}{f(r')} f'(r')$$

$$\frac{\varepsilon_0'_z}{\varepsilon_{0z}} = \frac{\mu_0'_z}{\mu_{0z}} = \frac{f(r')}{r'} f'(r')$$

In that layer, the object of ε_0 and μ_0 is optically cancelled. Thus, the object also becomes invisible to any incident waves.

Turning a cloak on and off with a switch requires a medium that can be ‘activated’ as well as ‘deactivated’. Ideally, the property needed is a material that when charged or otherwise ‘activated’, acquires negative permeability and permittivity but not when it is in its ‘passive’ state and has positive values for those features or possibly, one is positive and the other is negative.

There exist many papers concerning the practical and theoretic structures for MMs and their associated properties. Here, only a handful will be considered from the point of view of what may be practical, relatively inexpensive to manufacture and relatively simple. Therefore, work dealing with MM superconductors and MMs that rely on quantum dots and quantum-sized structures will not be considered.

Suppose we were inside our Eldridge and wished to keep the ship cloaked but also steer a course that required changes in direction say, every thirty seconds. Would we require a small uncloakable window through which we could visually observe the surroundings? The alternative to a window might be some optical periscope-like device protruding beyond the influence of the cloak but if that had a wide view angle, distortion might affect the judgment of distances and if not, the angle of view would be narrower. Alternatively, an uncloaked observation platform might suffice. The most practical alternative to these would be a few very small video camera not within the cloak linked to a large internal monitor providing the same view as that of a window. It seems to us that a window is a least elegant solution for accurate navigation when its comparative size with miniature cameras is considered.

Nevertheless, is it possible to switch off all or part of the cloak?

A 3D-array of toroidal solenoids displays a significant toroidal response that can be readily measured. This is in sharp contrast to materials that exist in nature, where the toroidal component is weak and hardly measurable. The existence of an optimal configuration, maximizing the interaction with an external electromagnetic field can be demonstrated. It is found that a characteristic feature of the magnetic toroidal response is its strong dependence on the background dielectric permittivity of the host material [61]. The first DC metamaterial was constructed that demonstrated a tuneable, highly anisotropic diamagnetic response, albeit at low temperature and operating in low applied magnetic fields. Most of the experimental difficulty lay in fabricating samples of sufficiently small size to fit inside a magnetometer. When this size constraint is lifted, structures of higher quality can be produced with greater ease [62]. The concept of a guided medium, based on subwavelength coaxial waveguides, decomposes incoming light waves into partial, propagating waveguide modes, that in turn re-assemble into propagating waves on the other side of the medium. Since the subwavelength waveguides do not experience frequency cut-off, a light wave is transmitted through the medium without loss of subwavelength resolution. Additional control over propagation in the waveguides can be obtained via active, non-linear nano-components. With nanoscopic, coaxial nanowaveguides, such as have recently been invented, operation of this medium can be extended to the visible frequency range [63]. Alternative designs make use of localized plasmon resonant metal nanoparticles or nanoholes in metal films. Following this approach and recently, a negative refractive index has been realized in the optical range. Numerical simulations show that a composite material comprising of silver strips and a gain providing material can have a negative refractive index of -1.3 and 100% transmission, simultaneously [64].

The proposed MMs consist of coated spheres embedded in a dielectric host. Simple design rules and formulas following the effective medium models are numerically and analytically presented. The revised Maxwell-Garnett effective medium theory enables a three-dimensional composite design of metamaterials through assembly of coated small spheres. The proposed approach allows for precise control of the permittivity and permeability and guides a facile, flexible and versatile way for the fabrication of composite MMs [65]. Theoretically, electromagnetic fields of certain frequencies can be parametrically shielded by a nonlinear left-handed material slab, where the permittivity and permeability are both negative. The skin depth is tunable, and even in the absence of material absorption, can be much less than the wavelength of the electromagnetic field being shielded [66]. Electrically tunable hybrid metamaterial consisting of special wire grid immersed into nematic liquid crystal is proposed. The plasma-like permittivity of the structure can be substantially varied due to switching of the liquid crystal alignment by external voltages applied to the wires. Depending on the scale of the structure, the effect is available for both microwave and optical frequency ranges [67].

A non-linear metamaterial is considered, composed of double split-ring resonators and wires where a varactor diode is introduced into each resonator so that the magnetic resonance can be tuned dynamically by varying the input power. At higher powers the transmission of the metamaterial becomes power-dependent and (demonstrated experimentally) power-dependent transmission properties and selective generation of higher harmonics are shown [68]. The nonlinear properties of a metamaterial sample composed of double-layer metallic patterns and voltage controllable diodes were experimentally investigated. Second harmonics and spectrum modulations were clearly observed in a wide band of microwave frequencies. That showed this kind of metamaterial is not only tunable by low DC bias voltage, but also behaves strong nonlinear property under a small power incidence [69].

A hybrid structure is presented in the waveguide environment that consists of a resonant magnetic material with a characteristic tuneable gyromagnetic response that is integrated into a complementary split ring resonator (CSRR) metamaterial structure. The combined structure exhibits a distinct hybrid resonance in which each natural resonance of the CSRR is split into a lower and upper resonance that straddle the frequency for which the magnetic material's permeability is zero. The designed structure demonstrates the potential for using a ferrimagnetic or ferromagnetic material as a means of creating a tunable metamaterial structure [70].

It would seem that at the present time, any 'window concept' rests on the de-tuning of a portion of our Eldridge from optical wavelength cloaking to some other frequency. However a switch to turn off a cloak is not out of the question. The most promising ideas for the purposes here is [69]. Most of the designs for tunable MMs are based on the mechanism that introduces some kind of controllable material or lumped elements to commonly used metamaterial unit cells, like SRRs, to construct an active metamaterial. In the optical frequency band, a photosensitive semiconductor is used. In the microwave frequency band, some ferroelectric/ferromagnetic composite metamaterial (CMM) are employed such as $Ba_{0.5}$ $Sr_{0.5}$ TiO_3 and $Y_3Fe_5O_{12}$. These MMs can be controlled by lights, high voltages or dynamic magnetic fields. In order to make a new active metamaterial that is more convenient to control, the nonlinear lumped element, varactor diode can be used that can be tuned by low direct current (DC) voltages. It demonstrates strong non-linear effects and under illumination of a low power incident EM wave, the phenomena of second-harmonic generation, signal demodulation and intermodulation are clearly observed from only one piece of the board made of such metamaterial. Such structure supports simultaneous electric and magnetic resonance and thanks to the existence of the diodes, will behave with both electric and magnetic nonlinear properties.

CONCLUSION

We have reviewed the relevant work listed in arXiv to the date of this paper concerning invisibility produced by metamaterials and like substances. We have also set out our thoughts concerning the cloaking of a ship.

APPENDIX

(Further considerations in no order of importance)

A1 Stealth

The U.S. Air Force is equipped with the stealth fighters and stealth bombers where, we are told, their shapes and coatings make them difficult to detect by radar. However, they are optically visible. It is not our intention to discuss this sort of stealth technology firstly because that remains top secret and secondly, it has almost nothing to do with the invisibility cloak.

A2 Quantum Physics of MMs

Tansformation media map electromagnetic fields in physical space to the electromagnetism of empty flat space and perform an active coordinate transformation of electromagnetism in physical space. Suppose a transformation medium acts on the electromagnetic field in the vacuum state. Although the medium appears to merely transform one nothing into another, it may cause for example, the Casimir force to be repulsive in left-handed transformation media, possibly even to the extent of levitating ultra-light metal foils. This follows from the quantum optics of spatial transformation media.

Cartesian coordinates x^i are given by the matrices

$$\varepsilon = \varepsilon' \frac{JJ^T}{\det J} \ , \qquad \mu = \mu' \frac{JJ^T}{\det J} \ , \qquad J^i_j = \frac{\partial x^i}{\partial x'^j}$$

The coordinates x^i and the scalars ε' and μ' are functions of some transformed coordinates x'^j. In regions where the coordinates agree, J reduces to the unity matrix; and so ε' and μ' directly describe the dielectric properties of the medium. The object is to show that the quantum optics of the general medium (of the three equations above) in physical space is mapped, in the primed coordinates, to quantum electrodynamics in the presence of a dielectric with ε' and μ', for the transformed vector potential **A’** = J**A.**

Write μ^{-1} and ε in component representation

$$\varepsilon = \pm \varepsilon' \gamma^{ij} \sqrt{\gamma} \ , \qquad \mu^{-1} = \pm \frac{\gamma_{ij}}{\mu' \sqrt{\gamma}} \ , \qquad \gamma = \det(\gamma_{ij}) \ ,$$

$$\gamma^{ij} = \sum_l \frac{\partial x^i}{\partial x'^l} \frac{\partial x^j}{\partial x'^l} \ , \qquad \gamma_{ij} = \sum_l \frac{\partial x'^l}{\partial x'^i} \frac{\partial x'^l}{\partial x'^j}$$

These are the constitutive equations of spatial transformation media expressed in terms of the spatial metric γ_{ij} being the matrix inverse of γ^{ij}. The $\pm$ sign distinguishes between right and left-handed coordinate systems where the Jacobian det J is positive or negative respectively. Left-handed systems thus correspond to left-handed materials where the eigenvalues of ε and μ are negative

Quantum electrodynamics in bi-anisotropic media is characterized by the vector potential **A** subject to the Coulomb gauge and the Maxwell equation $\nabla \times \mathbf{H} = \frac{\partial}{\partial t}\mathbf{D}$.

Express the Coulomb-gauge condition as

$$\pm\frac{\nabla\bullet\varepsilon\tilde{A}}{\sqrt{\gamma}}=\frac{1}{\sqrt{\gamma}}\sum_{ij}\frac{\partial\sqrt{\gamma}\ \gamma^{ij}\varepsilon'\tilde{A}_j}{\partial x^i}=0$$

that is the covariant form of the 3-D divergence if γ_{ij} is taken as the metric tensor of physical space. The physical Cartesian coordinates, x^i, appear as the back-transformed primed coordinates x'^j of a space where the metric tensor γ'_{ij} is the unity matrix, so the transformed space is flat. Consequently, $\nabla'\varepsilon\tilde{A}'=0$.

Regarding the Maxwell equation, apply the constitutive equations in SI units, **D** = $\varepsilon_0\varepsilon$ **E** and **B** = $\frac{\mu}{\varepsilon_0 c^2}$ **H** to the component representations and write the electric field in terms of the vector potential. Then, express the components of the curl in terms of the 3D Levi-Civita tensor for the spatial geometry characterized by the metric tensor and invert the constitutive equations for **B** using the representation of μ^{-1} in terms of the metric tensor. Lastly, apply the matrix γ^{ij} to **H** and represent **B** as the curl of **A** in covariant form. Equations combined are covariant, so in transformed space

$$\frac{1}{\varepsilon;}\nabla'\times\frac{1}{\mu'}\nabla'\times\tilde{A}'=-\frac{1}{c^2}\frac{\partial^2\tilde{A}'}{\partial t^2}$$

Consequently, the Coulomb-Maxwell equations for the transformed quantum field bA′ are the ones for flat space filled with a medium of permittivity ε' and permeability μ'. All the other building blocks of the quantum theory of light in transformation media, the Hamiltonian, the field commutator and the scalar product are naturally covariant and are therefore also mapped to flat space [11].

A3 The zeroth-order cylindrical wave

What would and what would not be cloaked once the material parameters are simplified? In the 2-D case, the cylindrical cloak has anisotropic spatially varying optical constants and some of the material parameters have infinite values at the interior surface of the cloak. To facilitate experimental realization at microwave frequencies, Schurig et al. simplified the material properties [12].

Yan et al claimed that, in comparison to the ideal cloak, the simplified cloak maintains power-flow bending with the penalty of non-zero reflectance at the outer interface. They provide a systematic theoretical study on simplified cylindrical cloaks and find that the bare device constructed using the simplified medium fails to be invisible with no spatial region that is electromagnetically isolated from the outside world. Hence, perfect hiding with such a simplfied cloak is not possible.

Consider a cylindrical cloak with its inner and outer boudaries positioned at r = a and r = b respectively. Both domains inside and outside the cloak are air. The structure is in general, a three-layered cylindrical scatterer. Referring to the layers from inside to outside as layers 1, 2 and 3, respectively with the **k** vector taken as perpendicular to the cloak cylinder axis. The EM wave is assumed to have TE polarization, that is, the electric field only exists in the z-direction. The cloak's cylindrical coordinates are the global coordinates and in a homogeneous material region (i.e. cloak interior and exterior) and the general solution is expressible in Bessel functions. Within the cloak medium, the general wave equation that governs the $\mathbf{E}_z$ field can be written as

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(\frac{r}{\mu_\theta}\frac{\partial E_z}{\partial r}\right)\right]+\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\mu_r}\frac{\partial E_z}{\partial\theta}\right)+k_0^2\varepsilon_z E_z=0$$

where k_0 is the free-space wave number, μ, μ_r and ε_r are the polarization-dependent permeability/permitivity profiles of the cloak.

Ideally, the cloak is designed to compress all fields within a cylindrical air region r < b into the cylindrical annular region a < r < b. A corresponding coordinate transformation leads to a set of anisotropic and spatially variant material constants in the cloak shell. Such an annular cylinder can provide perfect invisibility cloaking but it requires infinite values for optical constants at the cloak's inner boundary. To circumvent the fabrication difficulty, simplified parameters can be used for the 'Huang Cloak' [2] to achieve cloaking for TM waves for the same set of material parameters. However, the procedure for simplification of the material parameters adopted in [2] is questionable, as the derivation assumed beforehand that μ_θ is a constant and the invariant μ_θ can be taken out of the differential operator in the above equation. Therefore wave behavior within the cloak shell is altered compared to that in an ideal cloak.

Since the material parameters in that equation are azimuthally invariant (which is also true for the ideal parameter set), it can be decomposed such that the solution is $\exp(im\theta)$ with m as an integer. What results is a second-order homogeneous differential equation with two independent solutions expected. Fixing m gives rise to the generality $A_mQ_m+B_mR_m$ where A_m and B_m are constants. Q_m and R_m are functions of r. Now, valid field solutions in three layers can be described using the e-Hankel function of the second kind, which represents outward-travelling cylindrical waves. Two other terms in the 3rd layer are physically in correspondence to the incident and scattered waves. Hence, the scattering problem becomes the solution for the transmitted field and the scattered field subject to a given incidence.

The zeroth-order scattering coefficient is quite distinct from others due to the different governing wave equation. In fact, analytic solutions exists when m = 0, as the field in the cloak medium are Bessel functions. Using the analytic technique, the zeroth-order scattering coefficient is found to converge to a value other than zero but the scattering coefficient tends to converge to a value closer to zero when the order number m increases and numerical results show that the high-order scattering coefficients do not converge to zero even when the cloak wall is very thick.

Besides the requirement of zero scattering (invisibility), a device also need to possess a spatial region which is in complete EM isolation from the exterior. Therefore it is meaningful to know how much field penetrates into the simplified cloak subject to outside electromagnetic illumination. Again, the problem is studied by examining the individual cylindrical wave components separately. When m = 0, the amount of field transmitted into the cloak interior can be analytically derived as a function of b. The transmission is noticed to be oscillatory and converging to 1 as b increases.

When $m \neq 0$, FEM calculations show that the field inside the cloak is almost zero and the corresponding transmission coefficients are exclusively smaller than 0.005. This indicates that the contour r = a provides an insulation between its enclosed domain and the exterior domain, but only for all high-order cylindrical waves. Therefore, any objects placed inside the cloak are exposed to the zeroth-order cylindrical wave component. Conversely, the zeroth-order wave component of an EM source placed within the cloak (or scattered wave by objects inside the cloak) will transmit out. As a result, objects enclosed by a simplified cloak are detectable by a foreign detection unit.

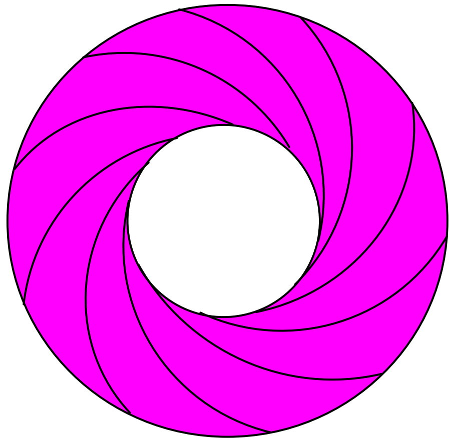
Theoretical cylindrical cloak designs with simplified parameters show that the scattering of such kind of cloaks can not be totally removed. In order to reduce the difficulties in the experimental realizations of the cloak but still keep good performance of invisibility, there is a feasible solution where the axial (z-direction) components of the material parameters are larger than 1 and spatially invariant. In theory, as long as the parameters of the metamaterials take the exact ideal forms a cloak with perfect invisibility can be realized. However, some small perturbations can still deteriorate the performance of the cloak. The scattering properties of the cloak with a perturbation at the inner boundary (cylindrical wave expansion method) indicates that both backward scattering and total scattering cross-section of this imperfect cloak can be suppressed by appropriately introducing loss. When a tiny change in ε_z is introduced but the refractive index remains the same, the cloak can still reduce the scattering cross-section omni-directionally, whilst keeping the impedance unchanged and can yield an even better performance in concealing objects from monostatic detection [13].

If a semi-analytical wave expansion method is employed using Mathieu functions to study the electromagnetic scattering properties of a 2-D elliptic cylindrical invisibility cloak, a small perturbation introduced into the ideal cloak can solve the boundary problem at the inner surface of the cloak shell.

Closed-form relations for the expansion coefficients of the resulting field inside the cloak can be deduced that are perfectly similar to those obtained for the circular cylindrical cloaks [14]. A cylindrical wave expansion method to study the electromagnetic scattering properties of a 2-D invisibility cloak can also be used. A near-ideal model of the invisibility cloak was set up to solve the boundary problem at the inner boundary of the cloak shell. By systematically studying the change of the scattering coefficients from the near-ideal case to the ideal one, it was confirmed that the cloak with the ideal material parameter is a perfect invisibility cloak. Due to the slow convergence of the scattering coefficients, a tiny perturbation on the cloak would induce a noticeable field scattering and penetration. The scattered and penetrated fields are dominated by zeroth-order cylindrical waves and this method can be extended to the 3-D spherical case [15].

A4 A cloak with oblique layers

A novel cloaking structure consisting of oblique alternating, rather than concentric alternating layers as drawn below



The optical properties of an oblique layered system with two kinds of isotropic materials can be described using the concept of transformation media as long as the thickness of the layers is much smaller than the wavelength. Once the connection with transformation media is established, the oblique layered system can serve as a universal element to build a variety of functional optical components such as wave splitters, wave combiners, one-dimensional cloaking devices and reflectionless field rotators.

The shifter, based on the oblique layered system, can be used as an element to build more complex devices, such as the one-dimensional cloaking device. The device has two thin slabs that are back-to-back, each with two shifters that shift light in opposite directions. These four shifters act together to open up a diamond-shaped cloaking region [16].

Essentially, all realizations of cloaking are so-called ‘reduced’ cloaks that are approximations of exact cloaking, as the exact implementation of perfect cloaks requires material properties that are too demanding for fabrication. ‘Reduced’ cloaks are not perfect and have some intrinsic scattering cross-sections. They show that the layered system configuration combined with the idea of transformation media gives the possibility of achieving a perfect rotation cloak that does not require any ‘reduction’, and only two kinds of isotropic materials are required.

Although there are many kinds of transformation media designed for novel applications, only a non-magnetic reduced cloak has been designed precisely using the concentric layered system. However, non-magnetic reduced cloaks demand infinitely many kinds of isotropic materials due to the position dependence of the permittivity tensor, making it very complex to realize.

What if the rotation cloak is embedded inside another medium? Suppose that two materials with suitable permittivities can be found. The materials can then serve as the materials to build the rotation cloak. In this case, rotation cloaking on a broad band can be achieved because all the permittivities are now larger than one and dielectrics can be used that have little dispersion and absorption.

Broad band functionality is specific to the rotation cloak that does not have singularities within the transformation media context. The working frequencies of the invisibility cloak (either ideal or reduced) cannot achieve broad band operation in this manner because a singularity exists in the permittivity tensor component in some positions.

A5 Arbitrary shaped cloak

A design for a dispersive cloak has been suggested based on the ideas in [16]. In contrast to the imperfect cloak, a concept of superscatterer was also proposed that can obtain a giant scattering cross-section beyond the device itself. If the inner core of the concentrator is replaced with a perfect electrical conducting cylinder, the concentrator can be employed as a device to change the scattering cross-section of the PEC cylinder. Its functionality is between the imperfect cloak and the superscatterer. In addition, a cylindrical cloak of arbitrary cross-section is described that can be easily extend to the imperfect cloak, concentrator, and superscatterer. A kind of transformation media called the “PEC reshaper” plus an explicit design for that is proposed. The PEC reshaper can reshape the PEC boundary arbitrarily if the objective effective PEC cylinder shares a domain with the original PEC cylinder. For example, the effective PEC cylinder can be partially inside the device and partially outside the device [17].

For an arbitrary shaped cloak, depict two boundaries, set the boundary conditions and then solve Laplace’s equations to obtain the deformation field inside the cloak layer. The material parameters of the cloak can be calculated numerically by the principle stretches evaluated from the deformation field. In order to check the performance of the designed cloak, the cloak device is illuminated by a plane electromagnetic wave with help of software [4]. The constructed arbitrary cloak in [4] did not disturb the incident waves and can shield an irregular obstacle from detection. For the physical realization, one can easily retrieve the material parameters of the arbitrary cloak from the deformation field. The proposed method can also be used to design arbitrary cloaks with many separated embedded obstacles by using the same system and perfect invisibility is still achieved even if the embedded obstacles are separated without disturbing the outside fields.

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A6 Frequencies

The design, fabrication, and characterization of a metamaterial absorber, resonant at terahertz frequencies and experimentally demonstrated was presented in [18] having an absorptivity of 0.97 at 1.6 terahertz. Importantly, this free-standing absorber is only 16 microns thick resulting in a highly flexible material that operates over a wide range of angles of incidence for both transverse electric and transverse magnetic radiation.

The design is on a freestanding, highly-flexible polyimide substrate 8μm thick that enables its use in non-planar applications as it can easily be wrapped around objects as small a 6 mm in diameter. In addition, it was demonstrated, through simulation and experiment that this metamaterial absorber operates over a very wide range of angles of incidence for transverse electric (TE) and transverse magnetic (TM) configurations. Finally, the bottom layer of the absorber consists of a continuous metal film which simplifies the fabrication in that, for this two layer structure, precise alignment between the layers is not required.

A7 Bandwidth

Apparently, transformation media equations can only be realized in one single frequency, and cannot be extended to a finite bandwidth (no matter how small the bandwidth) even if r_0 is allowed to be a function of frequency. Changing the material to the ‘reduced’ form will not help, since it has the same refractive index as an ellipsoid. In order to construct a cloak that works for a range of frequencies, further modifications are required. In fact, the invisibility condition sets a limit on the band width of the type of ‘reduced’ transformation media. The physical meaning of this condition is that there must be more tolerance of r_0[19].

A8 Singularities

A big problem in the full-parameter cloak is that some parameter components exist singularly on the inner boundary of the cloak that makes the full cloak difficult to achieve even using metamaterials. Elliptical cloaks have been designed and investigated in different coordinate systems. However, the cloak parameters are fully anisotropic and some components still have singular points on the inner boundary. In principle, the circular and elliptical cloaks try to crush the cloaked objects to a point that results in the singular parameters. In view of the difficulty to realize the full-parameter cloak and the imperfection of reduced parameter cloak, a recently published theory has suggested a carpet cloak that can hide any objects under a metamaterial carpet. Different from the complete invisible cloak, the carpet cloak crushes the hidden object to a conducting sheet. The great advantage of the carpet cloak is that it does not require singular values for the material parameters. However, the carpet cloak can only hide objects placed under a conducting plane, and cannot hide objects in free space. The elliptical invisible cloak using the coordinate transformation in classical elliptical-cylindrical coordinate system, does not shrink the cloaked object to a point, but crushes the object to a line segment that avoid any singularities in the constitutive parameters. Closed-form formulations are derived for both permittivity and permeability tensors [20].

A9 Field Rotation

For cloaking proposed by Pendry, the mapping is from a point to a circle while for the concentrator, the mapping is from a circle to another circle. The rotation coating is a transformation medium that performs a rotation of wave fronts. From basic transformation media theory, the Jacobian transformation matrix between the transformed coordinate and the original coordinate gives the associated permittivity and permeability tensors of the transformation media that rotates for the inner cylinder ($r = a$). The rotational angle is reduced to zero as the radius approaches $r = b$. Defining an ancillary anglet, the tensor components can be rewritten. A TE polarization full wave simulation at 1GHz shows that the plane wave changes its direction inside the enclosed domain and the energy inside and outside the rotation coating can flow in the opposite direction. Some turbulence-like pattern in the coating region occurs and in the extreme case, the incoming plane wave splits into two sets. Inside the rotation coating, one set has initially a faster phase velocity then slows down, while the other set has initially a slower phase velocity then speeds up. For other proscribed values, each set of rays goes around the origin many times and the rotation coating itself is not detectable to a far field observer.

Putting a scatterer in the inside domain, what would be seen in the outside world? When an object is placed inside the rotation coating, an observer in the outside world would see a rotated image from itself around the origin. Similarly, the image of the scatter outside is rotated from itself around the origin when observed from the inside. Observers inside and outside the rotation coating can communicate with each other, but the information is “rotated” [21].

A10 Casimir

Consider the resonant CP potential of an excited atom in front of meta-material half space. For long distances, the potential exhibits attenuated oscillations whilst close to the surface the potential becomes attractive or repulsive, depending on whether the absolute values of permittivity and permeability are larger or smaller than unity. The potential of an excited atom placed near a weakly absorbing left-handed planar superlens with a perfect mirror at the far end, shows that an enhanced superlens attraction away from the surface is genuinely due to the left-handed properties of the lens, while the potentials of right-handed materials closely resemble those of respective half spaces without the additional mirror [22].

How does the quantum physics of MMs modify the zero-point energy of the electromagnetic field and the resulting mechanical force of the quantum vacuum, the Casimir force? The Casimir force of two conducting plates may turn from attraction to repulsion if a perfect lens is sandwiched between them. For optical left-handed metamaterials, this repulsive force of the quantum vacuum may levitate ultra-thin mirrors. The usually attractive vacuum forces, the Casimir and the related van der Waals force are significant on the length scale of nanomachines so ideas for manipulating vacuum forces may find applications in nanotechnology [23].

A11 Optical and Near Optical Wavelengths

Resonant dielectric rods can be conceptually replaced by radiating electric and magnetic dipoles whose explicit expressions are derived in terms of the scattering matrix. These microscopic expressions are then used to compute the photonic band structure of a macroscopic square array of rods. The electric and magnetic activities of the rods are responsible for the opening of photonic band gaps and the creation of left-handed dispersion curves. The resonance phenomenon being an intrinsic property of the rods, it is possible to reproduce these effects at different frequencies. This is shown by the numerical demonstration of a true left-handed behavior at optical frequencies in a MM based on silicon rods [24].

Two metamaterial waveguides, operating in the mid-IR, would display negative refraction. The first waveguide is a metallic strip incorporating quantum wells, whereas the second is a dielectric waveguide which incorporates quantum wells. The ISBT of incorporated quantum well induces a region of negative group velocity equivalent to a negative refractive index. The effect is strongly dependent upon the 2-D electron density in the quantum well, and as such, one should be able to pump electron into the wells (optically or electrically) and thereby make the waveguide into an active component. The predicted losses in the negative index waveguides are small compared to other proposed schemes (high FOM values). Also, a more complicated quantum well structures could be used to induce gain (or even gain without population inversion) into the medium, thereby overcoming the losses [25].
A design was demonstrated for an optical cloak based on coordinate transformation. The non-magnetic nature of the design eases the pain of

constructing gradient magnetic MMs in three-dimensional space, and therefore paves the way for the realization of cloaking devices at optical frequencies. The proposed design can be generalized to cloaks with other metal structures, such as chains of metal nanoparticles or thin continuous or semi-continuous metal strips. It can be also adopted for other than the optical spectral ranges, including the infrared and the microwave. The achievable invisibility with the proposed cloak is not perfect due to impedance mismatch associated with the reduced material specifications and the inevitable loss in a metal-dielectric structure. Moreover, any shell-type cloak design can work only over a narrow frequency range, because the curved trajectory of light implies a refractive index of less than 1 in order to satisfy the minimal optical path requirement of Fermat's principle, whilst any metamaterial with $n < 1$ must be dispersive to fulfill causality [3].

A new type of cloak was discussed. It gives all cloaked objects the appearance of a flat conducting sheet. It has the advantage that none of the parameters of the cloak is singular. The cloak is designed to mimic a flat ground plane and the involved coordinate transform induces no singular values in the material profile. The quasi-conformal map is the optimal coordinate transform that can minimize the anisotropy of the physical medium. The small anisotropy is neglected so that the cloak can be synthesized by only isotropic dielectrics and it can relieve the loss issue by avoiding the usage of resonating elements. The quasi-conformal map should also be a convenient technique for both polarizations as well and in other applications in transformation optics [26].

It has been demonstrated that perfect invisibility from electromagnetic cloaks is only available for lossless metamaterials and within an extremely narrow frequency band [27].

A12 Reflections and Time Delay

Together with time delay, reflection waves are a distortion that prevents achieving a perfect invisibility device. However, the proposed device in [28] shows that due to impedance matching of negatively refracting materials, the reflection should be close to zero. This finding, together with the zero time delay, strongly indicates that perfect invisibility is possible in isotropic media. However, in Leonhardt's paper, "Notes on Conformal Invisibility Devices" (arXiv:phys/0605227v2), he writes "As a consequence of the wave nature of light, invisibility devices based on isotropic media cannot be perfect. The principal distortions of invisibility are due to reflections and time delays... " In another paper co-authored by Leonhardt (Hendi, Henn and Leonhardt), "Invisibility devices exploit ambiguities in the inverse scattering problem of light in media. We show that scattering is a tomographic projection: the integrated scattering angle is a projection of a scattering function onto the impact parameter. This function depends on the potential, but may be multi-valued, allowing for ambiguities where several potentials share the same scattering data. In addition, multi-valued scattering angles also lead to ambiguities. We apply our theory to show that it is in principle possible to construct an invisibility device without infinite phase velocity of light."

The transformation optical structures proposed to date, such as electromagnetic invisibility cloaks and concentrators, are inherently reflectionless and leave the transmitted wave undisturbed. The class of transformation optical structures can be expanded by introducing finite, embedded coordinate transformations that allow the electromagnetic waves to be steered or focused [29]. With numerical simulation confirmation, the necessary and sufficient condition for a reflectionless transformation medium in an isotropic and homogenous surrounding medium was established. The geometrical expression of this condition is that the transformed coordinates of the boundary can be the same as the original coordinates only by rotation and displacement of the coordinates. In a general sense, this condition is equivalent to the condition that the boundary coordinates are kept the same before and after transmission [30]. An electromagnetic cloak using high-order transformation functions to create smooth rather than discontinuous moduli at the outer boundary of the cloaking shell was proposed. By this approach, the undesired reflection from the system is completely eliminated within the limit of geometric optics, even for cloaks using non-magnetic materials. This scheme was applied to a non-magnetic cylindrical cloak and demonstration found that the scattered field from the outer boundary was reduced by almost an order of magnitude in a cloak with optimal quadratic transformation comparing to that with the usual linear compression [31].

Invisibility cloaks with arbitrary shape constructed by general coordinate transformations were confirmed to be perfectly invisible to the external incident wave. At the exterior boundary of the cloak, it was shown that no reflection was excited even though the permittivity and permeability did not always have a perfect matched layer form. At the inner boundary, no reflection was excited either and no field could penetrate into the cloaked region. However, for the inner boundary of any line transformed cloak, the permittivity and permeability in a specific tangential direction were always required to be infinitely large. Furthermore, the field discontinuity at the inner boundary always existed, the surface current being induced to make this discontinuity self-consistent. For a point transformed cloak, it did not experience such problems. The tangential fields at the inner boundary were all zero [32].

For the energy transport velocity distribution inside a 3-D ideal cloak, it is always smaller than the velocity of light in vacuum and has larger values along the incident direction in the line joining the origin of the cloak. The energy transport velocity near the inner boundary is much smaller or becomes zero at the inner boundary, and rays near the inner boundary take a much longer time to reach the other side of cloak. From the geometric optics viewpoint, a beam cutting across the origin of the cloak takes infinite time to pass through the cloak [33].

A13 Extending the Cloak

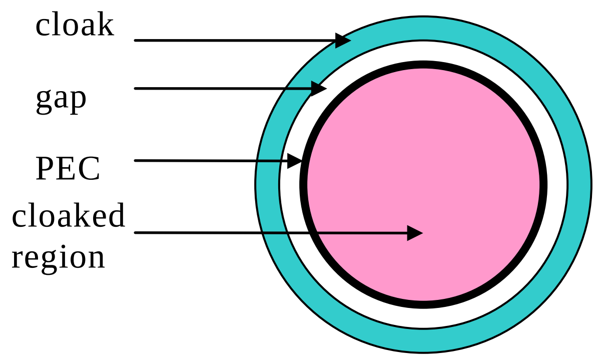
It is possible to have cloaking effects for an object that lies outside the cloaking shell. The origin of this external cloaking effect is the anomalous localized resonance of the coated cylinder that cancels any induced moment. To extend the cloak to hide another object that is external to the cloak itself, the cloak depends on the object, but the object can have arbitrary shape, so combining the concept of complementary media and transformation optics. A type of invisibility cloak that employs an “anti-object” embedded in a negative index shell can make an object that lays outside the cloaking shell invisible. The working principle can be described in two steps. Firstly, the object as well as the surrounding space is optically canceled by using a complementary media layer with an embedded complementary image of the object. Then, the correct optical path in the canceled space is restored by a dielectric core material. The total system is effectively equal to a piece of empty space fitted into the cancelled space [34].

A14 Surface Voltage

An infinite polarization of the material at the inner boundary of the cloak will induce electric and magnetic surface voltages that prevent all waves from going out. These peculiar surface voltages are rare in nature, but they do not violate Maxwell’s equations. The handedness of the polarization and phase information of the waves reflected from the inner boundary of the cloak are unchanged. The surface voltages due to an electric dipole inside the concealed region are equal to the auxiliary scalar potentials at the inner boundary that gain physical counterparts [35].

A15 Zeroth-order scattering

The problem of zeroth order scattering was found to be dominant among all cylindrical scattering terms. A type of simplified cloak with matched exterior boundaries was proposed [36]. The cloak uses non-magnetic material for the TM polarization and can function with a relatively thin thickness. A gap of free space is added to the design at the cloak's inner surface to eliminate the zeroth-order scattering (see diagram below), through the mechanism of scattering resonance. Due to destructive scattering resonance, complete cancellation of the zeroth-order scattering for certain thickness at the desired wavelength can be achieved.



It was found from simulations that the linear transformation based cloak introduced a level of back scattering similar to the one of a PEC cylinder without the cloak, causing the possibility of the object being detected. Such scattering can be significantly reduced by using the high-order transformation based cloak [37].

A16 Designs and Experiments

Designs of novel MMs consisting of quasi-static electric circuits with multiple independent current loops have been presented. These materials support a dark state leading to a phenomenon similar to electromagnetically induced transparency. The metamaterial exhibited a small transmission window with extremely low absorption losses and steep dispersion. No quantum mechanical atomic states were required in this metamaterial to observe the EIT. This could lead to 'slow light' applications from the microwave regime up to THz frequencies, where the structure can be most easily fabricated with more complex designs to deal with losses in MMs [38].

A metamaterial that acts as a strongly resonant absorber at terahertz frequencies can be achieved using a bilayer unit cell that allows for maximization of the absorption through independent tuning of the electrical permittivity and magnetic permeability. An experimental absorptivity of 70% at 1.3 terahertz was demonstrated [39].

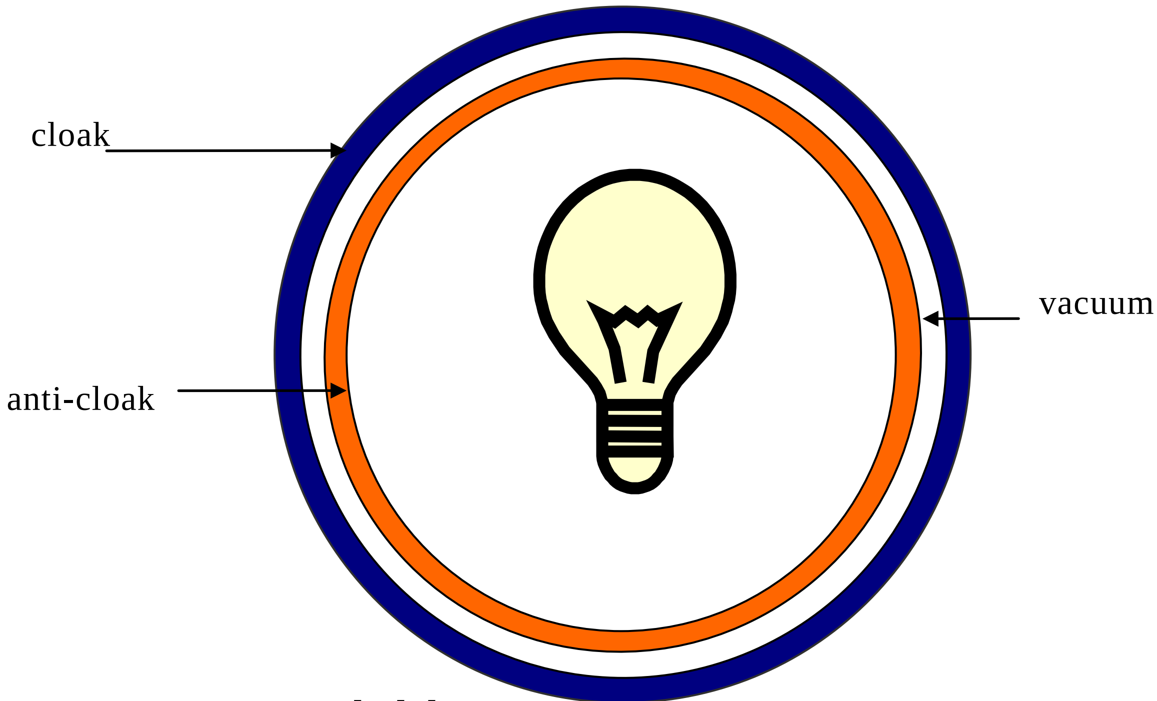
A class of resonant structures consisting of partial cylindrical shells of cloaking metamaterial with apertures that allow the penetration of incident irradiation into an interior resonant cavity were proposed. The study included micro-scale cavities with resonant frequencies in the optical range. In the resonant structures proposed, the resonant modes can be excited using free-space illumination [40].

A17 Anti-Cloaks

The invisiblity cloak cannot hide the enclosed domain if the inside domain has a shell of anti-cloak, being a transformation medium in itself. The anti-cloak region is an anisotropic negative refractive shell that is impedance matched to the cloak outside that has a positive refractive index. It is known that a negative refractive index medium ‘cancels’ the space of a positive index medium that has the same impedance. An heuristic way of understanding the operation of an anti-cloak is that it annihilates the functionality of the interior part of the invisibility cloak and effectively shifts the enclosed PEC region outwards to make contact with the outer part of the cloaking shell that is not ‘canceled’. That leads to a finite cross-section. Therefore, and very loosely speaking, a cloak inside a cloak is visible [41].

Elaborating on the concept of anti-cloak, the peculiar coupling effects that may be obtained by pairing a cloak and an anti-cloak separated by a vacuum layer, surrounding a dielectric or metamaterial cylinder were explored. It was shown that, depending on the constitutive parameters of the inner cylinder, four distinct (including DPS and SNG) configurations for an anti-cloak are possible that can relax some of the practical feasibility limitations of the DNG anti-cloak introduced by Chen. Besides the potential application (as a cloak ‘countermeasure’ - Chen), scenarios are suggested for which a region of space may be cloaked, while maintaining the capability of somehow ‘sensing’ the outside field from the inside. [42].

In effect, the concept of an anti-cloak is simple in that what one device does, the other undoes and in the diagram below, the light bulb is visible. It is suggested by us that if the cloaked object emits light (like the light bulb below), it is a matter of straightforward calculations to determine whether such a cloaked light source would be visible or not. The incident wave would impact the inner surface of the cloak rather than the outer surface.



A18 Lenses and Slabs

Conventional optic imaging in the visible spectrum shows resolution in the order of one wavelength, hence the reason for the electron microscope. With the advent of MMs, Pendry proposed the possibility of the perfect lens where the lens material could be ‘tailor made’ for its purpose.
The perfect lens is a slab of MM with its structures much smaller than one wavelength, and is of negative refraction with linearity, homogeneity, without spatial dispersion and is isotropic – the ‘ideal’ requirements.

It has been suggested to use a multilayer stack of alternating negative index and positive index materials as the lens. This effectively reduces its thickness. However, the distance from the object to the lens plus the distance from the lens to the image equals the thickness of each layer. In the limit when the layer thicknesses approach zero, the resulting effective medium acts as a fiber-optic bundle, but one that acts on the near field. To optimize the resolution and minimize aberrations, one again ends up with the problem of minimizing $|\varepsilon+1|$ and/or $|\mu+1|$. If aberrations can be tolerated, only $\mathrm{Im}(\varepsilon)$ and/or $\mathrm{Im}(\mu)$ need to be minimized. This can be achieved on a finite bandwidth at the expense of some variation of $\mathrm{Re}(\varepsilon)$ and/or $\mathrm{Re}(\mu)$ [43].
The unfolded geometry of MMs can act as an impedance matched hyperlens, and as the loss in the lens goes to zero finite collections of polarizable line dipoles lying

within a critical region surrounding the hyperlens are shown to be cloaked having vanishingly small dipole moments. This cloaking occurs in folded geometry and the equivalent unfolded one and is due to anomalous resonance, where the collection of dipoles generates an anomalous resonant field that acts back on the dipoles to cancel the external fields acting on them [10].

The perfect lens is very well explained by these workers. For achieving invisibility, electromagnetic space does not cover the entire physical space, whereas for perfect lenses electromagnetic space turns out to be multi-valued: single points in electromagnetic space are mapped to multiple points in physical space.

Consider in Cartesian coordinates the transformation x(x') where all other coordinates are not changed. In the fold of the function x(x'), each point x' in electromagnetic space has three exact images in physical space. Electromagnetic fields at one of those points are perfectly imaged onto the others and the device is a perfect lens with experiments confirming sub-resolution imaging.

The perfect lens made by negative refraction is not the only case of perfect imaging. In 1854 Maxwell proposed a device called the fisheye. In Maxwell's fish-eye light goes around in circles in such a way that any point in the device is imaged to a partner point with infinite precision. In turns out that Maxwell's fish-eye is an example of a transformation medium, but one with non-Euclidean geometry. Here, physical space is not mapped to flat space, but to the surface of a sphere [44].

Consider a bi-layer slab structures obtained by coordinate-transforming a single slab in EM space. The tangential fields at the two boundaries of such a bi-layer structure had the same values that were independent of the background material or the existence of active sources involved in the transformation. The transformed active sources in the bi-layer radiated no EM fields into the outside background. If the bi-layer structure was put in a homogenous background, it operates as a perfect tunneling lens that perfectly relays EM fields from one boundary to the other [45].
In Cartesian coordinates, the beam width of the Gaussian plane waves can be either compressed or expanded by a slab of the transformation optical structure, while in cylindrical coordinates, the radiation of an omni-directional line source can be modulated to specified directions through a cylindrical shell of the transformation optical structures. Both the EM field distributions and the power density flows through a 2-D full-wave numerical simulations have confirmed the performance of the beam modulating [46].

A transition layer for matching a slab of a backward-wave transmission-line network with free space and how the network impedance can be tuned to match that of free space was studied by simulation. The simulation results show that the layer can be used to match a backward-wave transmission-line network with free space and verify the negative refraction on the interfaces between free space and the network [47].

A19 Antennae and Waveguides

Transformation media can make a superantenna that is either completely invisible or focuses incoming light into a needle-sharp beam. The idea is based on representing three-dimensional space as a foliage of sheets and performing two-dimensional conformal maps on each sheet. This device is either invisible or it condenses a cross-section of light into a single point. Ideally, the condensed light would continue to propagate as a needle-sharp beam in a given direction, but there might be practical limitations to this behavior even though it may have unique three-dimensional properties, because it changes the topology of space. The device transforms the straight light rays of electromagnetic space into appropriately curved rays in physical space. Moreover, it not only transforms rays, but complete electromagnetic waves. Most non-trivial applications of transformation media use topology transformations such as spaces with holes in the case of invisibility, multivalued spaces for perfect lensing, topological bridges for electromagnetic wormholes and space-time horizons for artificial black holes [48].

The contra-directional coupling between a left-handed monomode waveguide and a right-handed monomode waveguide was studied using complex plane analysis. Light was shown to rotate in this lamellar structure forming a very exotic mode which was called a light wheel. The light wheel can be excited using evanescent coupling or by placing sources in one of the waveguides. This structure can thus be seen as a new type of cavity being a way to suppress the guided mode of a dielectric slab [49].

Devices that bend the propagation direction and squeeze confined electromagnetic fields have been described. Two approaches in non-magnetic realization of these structures were examined. The first was based on using a reduced set of material parameters and the second, on finding non-magnetic transformation media. It is shown that transverse-magnetic fields can be bent or squeezed to an arbitrary extent and without reflection using only dielectric structures [50].

Hawking showed that black holes can evaporate by emission of thermal radiation but later, a more general effect was given by Unruh. He showed that for an accelerating observer, vacuum should look like a bath of thermal radiation with temperature $T_U = \frac{\hbar a}{2\pi k c}$, where a is the acceleration of the observer. The Hawking temperature may be obtained from this by substitution of the free fall acceleration near the black hole horizon. As has been shown by Unruh, the proper choice of the electromagnetic eigenmodes, and hence the proper choice of the electromagnetic vacuum in a reference frame moving with constant acceleration ‘a’ should be done based on the normal-mode solutions in Rindler coordinates. Unruh showed that a specific coordinate transformation converts the Rindler metric into the Minkowski metric. The vacuum state that may be constructed based on the normal Minkowski eigenmodes looks like a thermal bath with temperature T_U. As a result, any detector moving through Minkowski vacuum with acceleration ‘a’ measures its Unruh temperature. While well-established theoretically, the Hawking effect and the Unruh effect are difficult to observe because an observer accelerating at a = 9.8 ms^{-2} should see a vacuum temperature difference of $4x10^{-20}$ K.

A metamaterial waveguide arrangement may lead to experimental observation of the more general Unruh effect. In principle, the metamaterial waveguide design may be approximated by a tapered optical waveguide. The MM optical waveguide geometry may produce an accelerated motion of a collection of photons (laser light) launched into it that behave as massive quasi-particles with internal degrees of freedom. The role of these internal degrees of freedom is played by the transverse modes of the metamaterial waveguide and an ensemble of these quasi-particles may be used as a test body to measure the Unruh temperature [51].

Not only the permittivity and permeability tensors of the media, but also the sources inside the media will take another form in order to behave equivalently as the original case. A source of arbitrary shape and position in the free space can be replaced by an appropriately designed metamaterial coating with current distributed on the inner surface and would not be detected by outside observers because the emission of the source can be controlled at will [52].

A20 Aharonov-Bohm Effect and Making Black Holes

Cloaks and hyperlenses work with excluded regions in physical space or folds in electromagnetic space. The optical Aharonov-Bohm effect arises when physical space is multi-valued, but the medium is single-valued and hence physically allowed. The effect can be demonstrated with light passing through a water vortex or with slow light in Bose-Einstein condensates. The optical Aharonov-Bohm effect is an example of a transformation medium that mixes space and time but the medium is stationary.

Consider a transformation in cylindrical coordinates and deduce that the medium is isotropic, has a refractive index and is magneto-electric. One factor corresponds to a fluid vortex with velocity profile with the constitutive equations of moving media in lowest relativistic order. Normally, a moving medium Fresnel-drags light, but in the case of a vortex, light rays follow straight lines because the transformation changes only time. Then, straight rays in electromagnetic space are mapped onto straight lines in physical space. In the Aharonov-Bohm effect electron rays are not bent by a magnetic vortex, but they develop a phase slip in the direction of incidence. In the other case, when the light has passed the vortex, the time change in the transformation results in a phase slip depending whether the light propagates with or against the current. Physical spacetime is multi-valued, with a branch cut in the direction of incidence, resembling the infinitely sheeted Riemann surfaces around a logarithmic branch point, but the medium is single-valued and has the simple physical interpretation of a moving fluid forming a vortex and moving isotropic media generate the effective spacetime geometry.

Consider an effectively one-dimensional situation where the medium is moving in x-direction and the electromagnetic field varies along the x-axis with field vectors pointing orthogonally to x. An impedance-matched medium with a refractive index and velocity is described by the inverse metric tensor and the velocity and the index can vary. This effective geometry is generated from empty space by a coordinate transformation if fields vary in x-direction. Wave packets are superpositions of waves propagating in positive or negative direction as functions of either $t+$ or $t-$. Substituting for $t\pm$ the representations as functions of ct and x, electromagnetic waves in physical space are obtained. The transformation describes the relativistic addition theorem of velocities for the medium and light propagating in positive or negative direction. When the velocity of the medium reaches the speed of light in the medium, the integral in $t\pm$ develops a logarithmic singularity. Light propagating against the current freezes with exponentially increasing oscillations at a horizon. This horizon is completely analogous to the event horizon of the black hole if the light is escaping from a superluminal region and to a white hole if the light is attempting to enter a counter-propagating superluminal medium.

Horizons cut physical space-time into distinct regions without possible communication and correspond to disconnected branches of multi-valued electromagnetic space, covering it multiple times and this is a magneto-electric analogue of an event horizon [5].

A21 Fluids

Acoustic stealth can be achieved using the concept of domain transformation. The fluid in the transformed region exactly replicates the acoustical properties of the original domain. The most general class of material that describes both the mimic and the mimicked fluids is defined as an acoustic metafluid. A general procedure for mapping/transforming one acoustic metafluid to another is described [53].

Transformation based invisibility cloaks applied to certain types of elastodynamic waves in structural mechanics have received less attention than the electromagnetic types because the Navier equations do not usually retain their form under geometric changes. It is possible to design a cylindrical cloak for in-plane coupled pressure and shear elastic waves using a rank 4 elasticity tensor with 16 spatially varying entries which are deduced from a geometric transform that in the transformed coordinates is no longer symmetric, which is a necessary condition for the Navier equations. Density remains a scalar quantity in the transformed coordinates and the Navier equations retain their form under that transform [54].

A22 Non-Ideal Cloaks

In sharp contrast to the rather formal transformations of Maxwell's equations in Minkowskian form used in the original pioneering paper and all subsequent publications, a simpler form of the time-independent transformations can be employed for solving problems. Based on these results, a simple recipe for evaluating the screening efficiency of non-ideal cloaks and their scattering dipole moment can be calculated [55].

A23 Orthogonal Coordinates

Transformation optics in orthogonal coordinates can be used. The method can obtain the material parameters of the transformation media when the mapping is represented by the orthogonal coordinates. Two applications of the transformation optics were detailed with several examples, such as the elliptic cylindrical cloak, the bipolar cylindrical cloak, the rotation cloak, the PEC reshaper and the kissing cloak [56].

A24 Photonics

Photonic metamaterials are engineered nano-composite structures with optimized responses to electromagnetic fields at optical frequencies that lead to the super- and hyper-lens and invisibility cloaks. Most metamaterials are fabricated using prohibitively expensive nanolithogtaphy techniques and are available in a form of sub-wavelength thin films. Recent advances in non-lithographic fabrication include direct laser writing, epitaxy, self-organized growth, and electroplating. A self-standing 51μm thick three-dimensional metamaterial based on the network of silver nanowires in an alumina membrane is realizable. This constitutes the anisotropic effective medium with hyperbolic dispersion that can be used in sub-diffraction imaging or optical cloaks. Highly anisotropic dielectric constants of the material range from positive to negative, and the transmitted laser beam shifts both toward the normal to the surface, as in regular dielectrics, and off the normal, as in dielectrics with the refraction index smaller than one.

Commercially acquired self-supporting anodic alumina membranes with the dimensions equal to 1cm x 1cm x 51μm, hole diameter equal to 35 nm and the porosity equal to 15% were filled with silver via the electroplating technique. Membranes were almost completely filled with silver at the electrode side, and the concentration of silver reduced toward the opposite side. In the best samples, there was high uniformity of filling. The synthesized metamaterial was highly anisotropic, with ε changing sign from positive to negative at the wavelength of the discontinuity of the Brewster angle. This designed photonic metamaterial is the thickest reported in the literature, both in terms of its physical size, d = 51 mm, and the number of vacuum wavelengths, N = 61 at λ = 0.84 μm and N=81 at λ = 0.63 μm [57].

Metamaterials represent a periodic array of designer sub-wavelength particles tuned to resonate at a specific frequency-band. At present, however, we have only a very basic understanding of the effect which a finite size of a sample of a periodic photonic crystal or metamaterial has on the macroscopic properties such as refraction. Yet every finite dielectric object is a moderate-quality resonator whose eigenmodes form a virtual photonic lattice with its own angular band-gaps and preferred directions of propagation. This virtual lattice produces nontrivial real effects and that even a homogeneous dielectric resonator may refract negatively without either negative or periodically modulated permittivity/permeability. A simple way to control the period of this virtual photonic lattice is by varying the transverse dimension of the resonator [58].

A25 Ray Tracing

The material properties associated with a coordinate transformation can be used to perform ray tracing. Examples, of spherical and cylindrical cloaks are worked out in some detail. Some of the value in this effort is to provide independent confirmation that the material properties calculated from the transformation do indeed cause electromagnetic waves to behave in the desired and predicted manner. Eventually, this technique will become more accepted and independent confirmation will not be needed. One can see what the waves will do much more easily by applying the transformation to the rays or fields in the original space where the behavior is simpler if not trivial. However, one may still want to perform ray tracing on these media to see the effects of perturbations from the ideal material specification [59].

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